D-instantons and holography

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Outline

What is an instanton?

Classical solution of euclidean YM e.o.m.

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- Finiteness of action requires

$$A_\mu(x) \to U^{-1}(x)\partial_\mu U(x), \quad x \to \infty$$

- Such configurations classified by winding at infinity

$$\Pi_3(S_3^\infty) = \mathbb{Z}$$

- Each sector characterized by topological charge

$$k = -\frac{1}{16\pi^2}\int d^4x \,\mathrm{tr} * F_{\mu\nu}F_{\mu\nu}$$

What is an instanton?

Configuration which minimizes YM action

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- Non-vanishing instanton action $\Rightarrow$ non-perturbative effects

$$S_{\mathrm{cl}} = \frac{8\pi^2|k|}{g^2} \quad \Rightarrow \quad \langle \ldots \rangle_k \propto \mathrm{e}^{-\frac{8\pi^2|k|}{g^2}}$$

The 't Hooft solution ('76)

For gauge group $SU(2)$, at instanton number $k = 1$, the self-duality condition is solved by

$$A_\mu(x; x_0, \rho, \theta_a) = U^{-1}(\theta_a)\left(\sigma_{\mu\nu}\frac{(x-x_0)_\nu}{(x-x_0)^2+\rho^2}\right)U(\theta_a)$$

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- It will have a very natural realization in terms of D-branes

Instanton calculus

In a semiclassical approximation, the path integral splits in a sum over topological sectors

$$\langle \mathcal{O}_1 \dots \mathcal{O}_n \rangle \sim \sum_{k \in \mathbb{Z}} \mathrm{e}^{-\frac{8\pi^2 |k|}{g^2}} \int d\mathcal{M}_k \mathrm{e}^{-S_{\mathrm{mod}}(\mathcal{M}_k)} \mathcal{O}_1 \dots \mathcal{O}_n$$

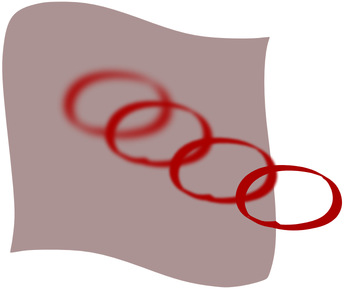
- Counting all configurations in each sector $\Rightarrow$ integration over moduli space
- Measure over moduli space ($\Leftarrow$ ADHM) $\Leftrightarrow$ moduli action $S_{\mathrm{mod}}(\mathcal{M}_k)$ given by evaluation on the instanton solution
- When fermions are present (e.g. SUSY), integration over fermionic moduli selects correlation functions with certain fermionic insertions at each k

Some uses of instantons

- Tunnelling effects in QM
- Perturbatively forbidden couplings in effective action
- Vacuum structure of SYM theories:
  - $\mathcal{N} = 1$ ADS superpotential
  - $\mathcal{N} = 2$ SW theory
- ...

D-branes and closed strings

- (p+1)-dimensional classical solutions of supergravity e.o.m.
- Sources for SUGRA fields ∼ massless spectrum of closed strings
- Tension T_p and R-R charge μ_p satisfying BPS condition



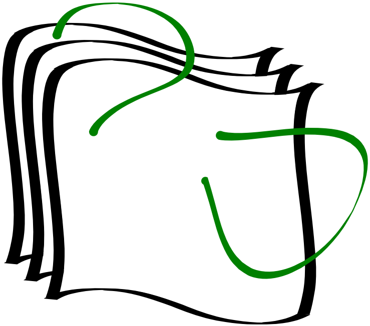
$$ds^2 = H(r)^{-\frac{1}{2}}\eta_{\alpha\beta}dx^\alpha dx^\beta + H(r)^{\frac{1}{2}}\delta_{ij}dy^i dy^j,$$

$$\mathrm{e}^\phi = H(r)^{-\frac{p-3}{4}}, \quad C_{(p+1)} = H(r)^{-1}dx^0 \wedge \ldots \wedge dx^p,$$

$$H(r) = 1 + \frac{\mu_p}{r^{7-p}}$$

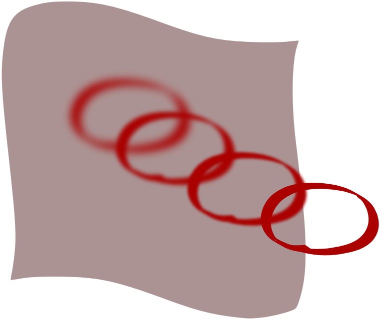
D-branes and open strings

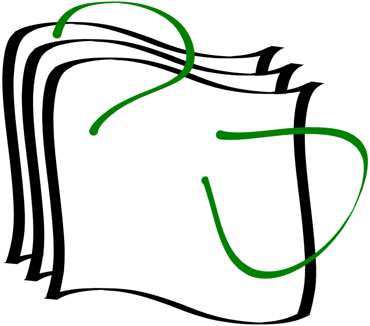
- Open strings have endpoints fixed on D-branes
- Worldvolume dynamics encoded in their spectrum
- Multiple coincident D-branes $\Rightarrow$ non-abelian structure



Massless spectrum comprises gauge bosons, scalars and fermions
$\Rightarrow$ (p+1)-dimensional non-abelian susy gauge theories

D-brane dynamics



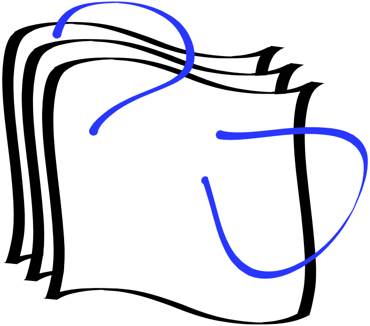


Taking into account both gravitational and gauge d.o.f.

$$S_{\mathrm{Dp}} = T_p \int d^{p+1}x e^{-\phi} \,\mathrm{tr}\, \sqrt{-\det(G + B + 2\pi\alpha' F)}$$
$$+ i\mu_p \int d^{p+1}x \,\mathrm{tr}\, e^{B+2\pi\alpha' F} \wedge \sum_q C_{(q)}$$

D-branes and gauge theory: an example

N D3-branes in flat $\mathbb{R}^{9,1}$



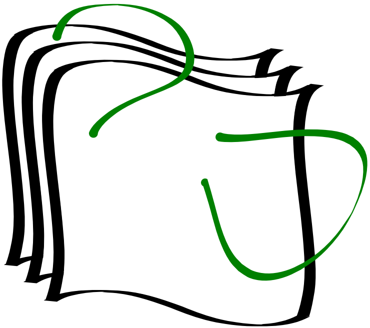
- Worldvolume is $\mathbb{R}^{3,1}$
- Open string massless spectrum (in the adjoint of $U(N)$)
  - gauge boson A_μ
  - 6 scalars ϕ_a
  - 4 fermions $\Lambda_{\alpha A}, \bar{\Lambda}^{\dot{\alpha} A}$

$\Rightarrow$ the system realizes 4-dimensional $\mathcal{N} = 4$ SYM with gauge group $U(N)$

D-branes and gauge theory: another example

N D7-branes in $\mathbb{R}^{7,1} \times \mathcal{T}_2$ projected by world-sheet parity $\times$ inversion on $\mathcal{T}_2$ $\Rightarrow$ unoriented strings

- Worldvolume is $\mathbb{R}^{7,1}$
- Orientifold projection $\Rightarrow$ $SO(2N)$ gauge group
- Displacement in $\mathcal{T}_2$ $\Leftrightarrow$ scalar vevs



$\Rightarrow$ the system realizes 8-dimensional $\mathcal{N} = 1$ $SO(2N)$ gauge theory

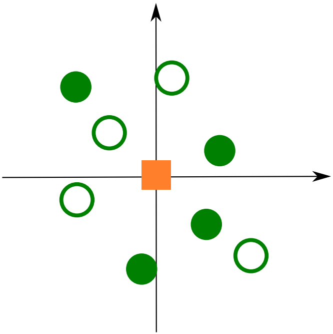
Looking at the gravity side

N D7-branes in $\mathbb{R}^{7,1} \times \mathcal{T}_2$ projected by world-sheet parity $\times$ inversion on $\mathcal{T}_2 \Rightarrow$ unoriented strings

- D7-branes emit closed strings $\Rightarrow$ fields propagating in $\mathcal{T}_2$
- Focus on dilaton and C_8 dualized into $C_0 \Rightarrow$ complex scalar

$$\tau = C_0 + \mathrm{i}e^{-\phi}$$

- O7-plane sources the same fields



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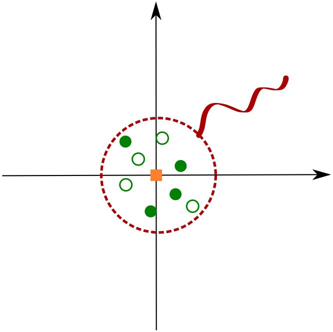

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$\Rightarrow$ classical axion-dilaton profile around an orientifold fixed point

$$\tau_{\mathrm{cl}}(z) = \frac{1}{2\pi\mathrm{i}} \sum_{i=1}^{N} \left[\log \frac{z - z_i}{z_0} + \log \frac{z + z_i}{z_0} \right] - 8 \log \frac{z}{z_0}$$

Holography and D-branes

- D-branes admit two a-priori different descriptions:
  - Their worldvolume supports gauge theories
  - They source non-trivial gravitational fields
- Can this correspondence be quantitatively established?

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- D-branes admit two a-priori different descriptions:
  - Their worldvolume supports gauge theories
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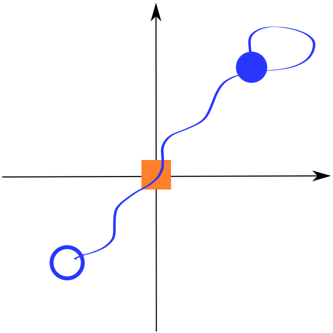
- Can this correspondence be quantitatively established?
- Holography: duality between (d+n)-dim gravity theories and d-dim gauge theories
- A possible realization of holography: probing the background geometry sourced by Dp-branes with other Dq-branes $\Rightarrow$ gauge couplings on Dq worldvolume $\Leftrightarrow$ gravitational background

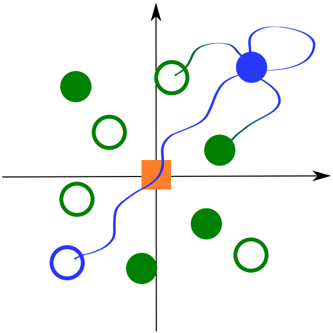
D3 probe in D7/O7 background I

- Consider a D3-brane in orientifold background $\Rightarrow$ worldvolume theory is $\mathcal{N} = 2$ 4d $Sp(1) \sim SU(2)$ SYM



D3 probe in D7/O7 background I

- Consider a D3-brane in orientifold background ⇒ worldvolume theory is $\mathcal{N} = 2$ 4d $Sp(1) \sim SU(2)$ SYM
- Open strings stretching between D3 and D7's give N fundamental matter hypermultiplets
- D7 gauge symmetry ⇔ D3 global symmetry $SO(2N)$



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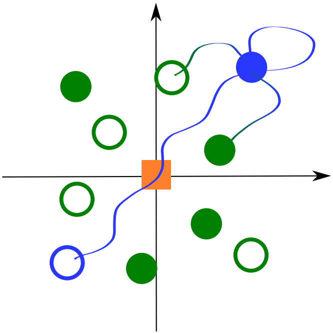
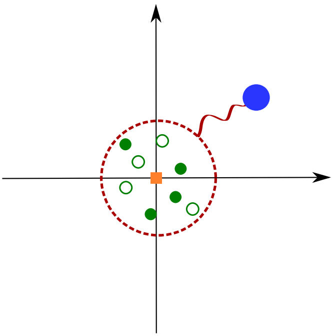
- Open strings stretching between D3 and D7's give N fundamental matter hypermultiplets
- D7 gauge symmetry $\Leftrightarrow$ D3 global symmetry $SO(2N)$
- If D3 brane is placed away from the origin, $SU(2)$ gets broken to $U(1)$ by adjoint scalar
- Moving D3 in $\mathcal{T}_2$ $\Leftrightarrow$ exploring Coulomb branch



D3 probe in D7/O7 background II

- From D3 action, $\tau \Leftrightarrow$ D3 gauge coupling
$$\int \tau \, \mathrm{tr}\, F^2$$
- $\tau_{\mathrm{cl}}(z)$ is field emitted by D7/O7 and felt by D3 placed at z
- If $m_i = \frac{z_i}{2\pi\alpha'}$ and $a = \frac{z}{2\pi\alpha'}$ indeed coincides with 1-loop coupling of 4d $\mathcal{N} = 2$ $SU(2)$ SYM with N hypers
- In the gauge theory, τ receives instanton corrections (Seiberg-Witten); how do they arise in the gravity dual?

$\Rightarrow$ D-instanton calculus



Building 4d gauge instantons I

Recall N D3-branes in Type IIB $\Rightarrow$ perturbative 4d $\mathcal{N} = 4$ $SU(N)$ SYM

- D3 action contains coupling to topological density

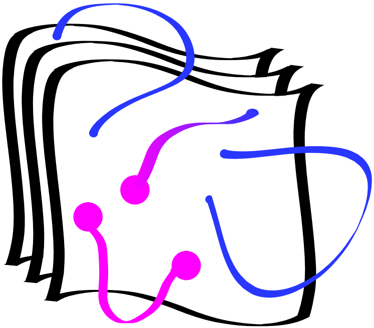
$$\int C_0 \,\mathrm{tr}\, F \wedge F \quad \left(\Leftarrow \int d^4x \,\mathrm{tr}\, \mathrm{e}^F \wedge \sum_q C_{(q)} \right)$$

- C_0 sourced by D(-1)-branes:
  - D(-1)'s are also solutions of Type IIB
  - D3/D(-1) bound state is still BPS $\Rightarrow$ stable configuration
  - Classical D(-1) action is $\frac{8\pi^2}{g_{\mathrm{YM}}^2}$

$\Rightarrow$ topologically non-trivial sectors of the gauge theory living on D3 worldvolume are realized by adding k D-instantons on top of it

Building 4d gauge instantons II

- New open strings: D3/D(−1) and D(−1)/D(−1)
- No momentum ⇒ moduli
- Massless spectrum ⇔ ADHM moduli
- Interactions give $S_{\rm mod}(\mathcal{M}_k)$ and ADHM constraints



From gauge to exotic instantons I

Localized configurations in effective 4d gauge theory can be realized in more general setups:

- D3-branes ⇒ D($n+3$)-branes filling 4d spacetime and wrapped on some n-cycles of the compact manifold: effective 4d field theory
- D(-1)-branes ⇒ E($n-1$)-branes wrapped on some n-cycles of the compact manifold: localized in 4d

Massless spectrum of open strings in these systems depends on configuration in the compact manifold ⇒ two main situations

From gauge to exotic instantons II

1. D$(n+3)$/E$(n-1)$ wrapped on the same cycles
   - String moduli ⇔ ADHM moduli
   - Moduli interactions give $S_{\text{mod}}(\mathcal{M}_k)$ and ADHM constraints

   ⇒ ordinary gauge instantons
2. D$(n+3)$/E$(n-1)$ wrapped on different cycles
   - Some moduli are missing
   - No ADHM constraints

   ⇒ exotic instantons, new effects absent in field theory

Exotic nature comes from having more than 4 ND directions ⇒ same features in 8d gauge theory with pointlike configurations:

D7/D(-1) in Type I' theory

Exotic instantons in 8d I

Recall D7/O7 system: effective 8d $SO(2N)$ gauge theory on D7 worldvolume

- D7/D(-1) system still BPS $\Rightarrow$ stable configuration
- D7 action contains a term coupling C_0 (sourced by D-instantons) and quartic topological invariant of 8d gauge field

$$\int C_0 \, \mathrm{tr}\, F \wedge F \wedge F \wedge F \quad \left(\Leftarrow \int d^8x \, \mathrm{tr}\, \mathrm{e}^{F} \wedge \sum_q C_{(q)} \right)$$

- On solutions with topological invariant k, D7 quartic action reduces to action of k D-instantons

$\Rightarrow$ D(-1)-branes added to D7 worldvolume represent instanton-like configurations of 8d field theory

Exotic instantons in 8d II

- Exotic vs gauge instantons:
    - $(-1)/(-1)$ sector analogous to ordinary case
    - $(-1)/7$ sector very different: no bosonic moduli
    - No ADHM constraints
- Very different in field theory, but on the same footing in string theory $\Rightarrow$ extrapolate D-instanton calculus prescription to exotic case

The D7-O7 system

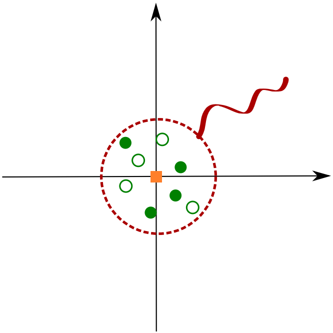
N D7-branes on top of O7-plane in Type I'

- 8d $\mathcal{N}=1$ $SO(2N)$ adjoint chiral superfield

$$M = m + \sqrt{2}\theta\Lambda + \frac{1}{2}\theta\gamma^{\mu\nu}\theta F_{\mu\nu} + \ldots$$

- Source for $\tau = C_0 + \mathrm{i}e^{-\phi} \to T$

$$T = \tau_0 + \sqrt{2}\theta\lambda + \ldots + 2\theta^8 \frac{\partial^4}{\partial z^4}\bar{\tau}$$



Classical τ profile (for $N=4$)

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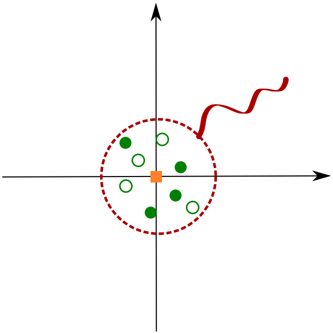


Classical τ profile (for $N = 4$)

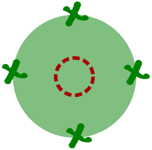
$$\tau_{\mathrm{cl}}(z) = -\frac{1}{2\pi \mathrm{i}} \sum_{l=1}^{\infty} \frac{(2\pi\alpha')^{2l}}{2l} \frac{\mathrm{tr}\, m_{\mathrm{cl}}^{2l}}{z^{2l}}$$

8d prepotential and source

Effective action from interaction between T and M

$$S_{\text{pert}}(M,T) = \frac{1}{(2\pi)^4}\int d^8x d^8\theta F_{\text{pert}}(M,T) + \text{c.c.}$$

$$F_{\text{pert}}(M,T) = 2\pi \mathrm{i} \sum_{l=1}^{\infty} \frac{(2\pi\alpha')^{2l-4}\,\text{tr}\, M^{2l}}{(2l)!} \frac{\partial^{2l-4} T}{\partial z^{2l-4}}$$



E.o.m and source for τ

$$S_{\text{bulk}} = -\frac{1}{2\tilde{\kappa}^2}\int d^{10}x \partial_M \bar{\tau} \partial^M \tau, \quad S_{\text{source}} = -\frac{T_7}{\tilde{\kappa}}\int d^8x J_{\text{cl}} \bar{\tau} + \text{c.c.}$$

$$\frac{\delta}{\delta \bar{\tau}}(S_{\text{bulk}} + S_{\text{source}}) = 0 \quad \Rightarrow \quad \Box \tau = J_{\text{cl}} \delta^2(z)$$

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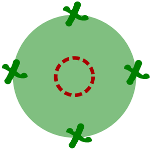


E.o.m and source for τ

$$J_{\text{cl}} = -2\mathrm{i}\sum_{l=1}^{\infty}\frac{(2\pi\alpha')^{2l}\,\mathrm{tr}\,m_{\text{cl}}^{2l}}{(2l)!}\frac{\partial^{2l}}{\partial z^{2l}}$$

$$\Rightarrow J_{\text{cl}} = -\frac{(2\pi\alpha')^4}{2\pi}\frac{\delta F_{\text{pert}}(M,T)}{\delta(\theta^8\bar{\tau})}\Big|_{T=\tau_0,M=m_{\text{cl}}} \equiv \bar{\delta}F_{\text{pert}}(M,T)$$

Non-perturbative extension

- D-instantons source topologically non-trivial field configurations on D7's

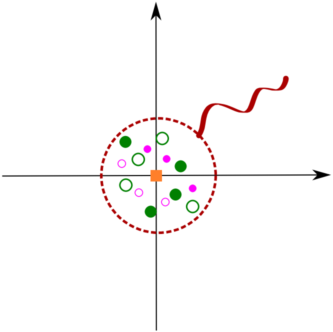
$$F(M,T) = F_{\text{pert}}(M,T) + F_{\text{n.p.}}(M,T)$$

- Non-perturbative corrections to the source ⇒ non-perturbative solution for τ

$$J_{\text{n.p.}} = \bar{\delta} F_{\text{n.p.}}(M,T)$$

- Non-perturbative prepotential from D-instanton calculus

$$F_{\text{n.p.}} = \sum_k \int d\widehat{\mathcal{M}}_k \, \mathrm{e}^{-S_{\text{inst}}(\mathcal{M}_k, M, T)}$$



The instanton action

Instanton action comes from interactions between moduli and M,T
$\Rightarrow$ disk diagrams with (part of) boundary on D(−1)'s

$$S_{\rm inst}(\mathcal{M}_k, M, T) = -2\pi k \tau_0 + S_{\rm mod}(\mathcal{M}_k, M) + S_{\rm mod}(\mathcal{M}_k, T)$$

- $-2\pi k \tau_0$ from pure k D(−1) disks
- $S_{\rm mod}(\mathcal{M}_k, M)$ exotic instanton action for D7/D(−1)

The instanton action

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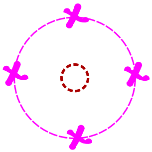
- $-2\pi k \tau_0$ from pure k D(−1) disks
- $S_{\text{mod}}(\mathcal{M}_k, M)$ exotic instanton action for D7/D(−1)
- New ingredient is $S_{\text{mod}}(\mathcal{M}_k, T)$ ⇒ compute mixed open/closed disk diagrams



$$\Rightarrow S_{\text{mod}}(\mathcal{M}_k, T) = -2\pi \mathrm{i} \sum_{l=0}^{\infty} \frac{(2\pi\alpha')^{2l}}{(2l)!} \,\mathrm{tr}\, \chi^{2l} \bar{p}^{2l}\, T$$

Computing the non-perturbative source

- Introducing $q = \mathrm{e}^{2\pi \mathrm{i} \tau_0}$,

$$J_{\text{n.p.}} = \bar{\delta} F_{\text{n.p.}}(M, T) = \sum_{k=1}^{\infty} q^k \bar{\delta} F_k$$

- $\bar{\delta} F_k$'s are computed in terms of $\bar{\delta}$-variations of usual instanton partition functions

$$Z_k = \int d\mathcal{M}_k \mathrm{e}^{-S_{\text{inst}}(\mathcal{M}_k, M, T)}$$

- From the explicit expression for $S_{\text{mod}}(\mathcal{M}_k, M, T)$

$$\bar{\delta} Z_k = 4\pi \mathrm{i} \sum_{l=0}^{\infty} (2\pi\alpha')^{2l} \bar{p}^{2l+4} Z_k^{(2l)}$$

$$Z_k^{(2l)} = \frac{1}{(2l)!} \int d\mathcal{M}_k \, \mathrm{tr}\, \chi^{2l} \mathrm{e}^{-S_{\text{inst}}} \Big|_{T=\tau_0, M=m_{\text{cl}}}$$

The 8d chiral ring and the n.p. source

- Same objects $Z_k^{(2l)}$'s appear in the computation of non-perturbative contributions to the chiral ring of the 8d gauge theory

$$\langle \mathrm{tr}\, m^J \rangle_{\mathrm{n.p.}}$$

- Explicit relation between $\bar{\delta} F_k$'s and chiral ring elements

$$\bar{\delta} F_k = 4\pi \mathrm{i} \sum_{l=0}^{\infty} (-1)^l \frac{(2\pi\alpha')^{2l}}{(2l+4)!} \bar{p}^{2l+4} \langle \mathrm{tr}\, m^{2l+4} \rangle_k$$

- Non-perturbative source for τ is expressed in terms of non-perturbative 8d chiral ring

$$J_{\mathrm{n.p.}} = -2\mathrm{i} \sum_{l=0}^{\infty} (-1)^l \frac{(2\pi\alpha')^{2l}}{(2l)!} \bar{p}^{2l} \langle \mathrm{tr}\, m^{2l+4} \rangle_{\mathrm{n.p.}}$$

The exact τ profile

- Solving the e.o.m. with $J_{\text{n.p.}}$ yields

$$\tau_{\text{n.p.}}(z) = -\frac{1}{2\pi\mathrm{i}}\sum_{l=1}^{\infty}\frac{(2\pi\alpha')^{2l}}{2l}\,\frac{\langle \operatorname{tr} m^{2l}\rangle_{\text{n.p.}}}{z^{2l}}$$

- Exact result given by replacing $\operatorname{tr} m_{\text{cl}}^{2l}$ in the perturbative solution with the full quantum correlator $\langle \operatorname{tr} m^{2l}\rangle$

$$\tau(z) = \tau_0 - \frac{1}{2\pi\mathrm{i}}\sum_{l=1}^{\infty}\frac{(2\pi\alpha')^{2l}}{2l}\,\frac{\langle \operatorname{tr} m^{2l}\rangle}{z^{2l}}$$

- Identyfing $a = \frac{z}{2\pi\alpha'}$ as the position of a D3 probe, $\tau(a)$ coincides with the exact coupling of $\mathcal{N} = 2$ $SU(2)$ 4d SYM given by SW theory

$\Rightarrow$ Non-perturbative realization of gauge/gravity duality

Ongoing work and publications

Ongoing work

- Using D-instanton calculus to compute non-perturbative corrections to other gravity fields emitted by D7/O7 system
- Apply same ideas to a different system which has the same gauge dual, namely fractional D3-branes in $\mathbb{R}^4 \times \mathbb{C} \times \mathbb{C}/\mathbb{Z}_2$

Publications

- Billò, Frau, Giacone, Lerda: *Holographic non-perturbative corrections to gauge couplings*, JHEP 1108 (2011) 007, `arXiv:1105.1869 [hep-th]`
- Billò, Frau, Giacone, Lerda: *Non-perturbative gauge couplings from holography*, `arXiv:1201.4231 [hep-th]`