After 2 years of PhD

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February 13, 2012

Contents

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Compactification of Closed Bosonic String

$$X^{25}(\sigma+\pi,\tau) = X^{25}(\sigma,\tau) + 2\pi RW\,, \qquad W \in \mathbb{Z}$$

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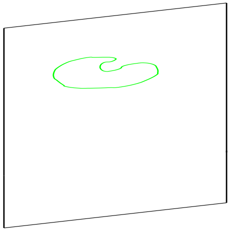

Transformation $W \leftrightarrow K\,, \qquad R \leftrightarrow \frac{1}{R}$:

- leaves M^2 invariant
- changes compactification radius

$\Rightarrow$ Target Space Duality

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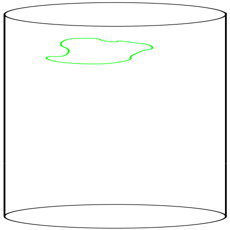

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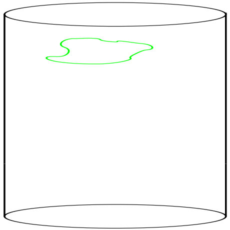

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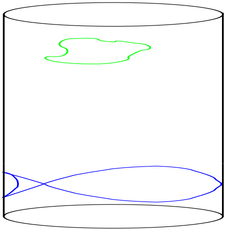

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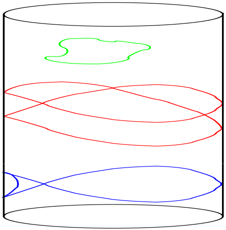

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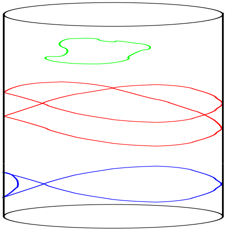

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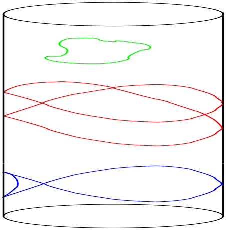

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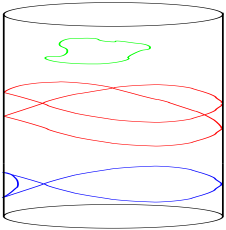

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- action invariant under $y \longrightarrow y + \lambda$

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- $G_{00} \longrightarrow \frac{1}{G_{00}}$: Target Space Transformation $\longrightarrow$ T-duality

Review 3: Developments

- T-duality construction for abelian isometries $x \to x + \lambda$

T.H. Buscher, *Phys. Lett.* **B 201** (1988)

- T-duality construction for non-abelian, linear isometries $x \to x + \lambda x$

X.C. de la Ossa, F. Quevedo – hep-th/9210021

- Dual Superconformal Symmetry: *CFT* counterpart of *AdS* Self-T-duality

Self-T-duality of $AdS_5 \times S^5$: bosonic$\oplus$fermionic T-duality

N. Berkovits, J. Maldacena – arXiv:0807.3196
N. Beisert, R. Ricci, A.A. Tseytlin, M. Wolf – arXiv:0807.3228

- Limit of $AdS_5 \times S^5$: purely fermionic coset σ-model

N. Berkovits – hep-th/0703282

We Need an Analysis of Quantum Fermionic T-duality

Outlook

Playground:

- Coset spaces technique for σ-models
- $OSp(n|m)$ supergroup
- Examples and general case $\frac{OSp(n|m)}{SO(n) \times Sp(m)}$

Fermionic T-duality:

- Simple examples $\longrightarrow$ Obstructions
- New methods
- Dualization of general *OSp* fermionic model

Quantum Analysis:

- BFM method
- 1- and 2-loop self energy computation

Fermionic Coset Spaces

Generalities

- Coset group $G/H \longrightarrow$ algebra $\mathfrak{g}$, generators $K^M = \left\{ T^a \in \frac{\mathfrak{g}}{\mathfrak{h}} \,,\, H_i \in \mathfrak{h} \right\}$
- Coset representative: choice of X_A in $g = e^{K^A X_A}$
- Vielbein: V^a: $g^{-1}\mathrm{d}g = V^a T_a + \text{H-connection}$
- Sigma model action: $S = \int_\Sigma \mathrm{d}^2 z \sqrt{-\eta}\eta^{\mu\nu} \; V^a_\mu V^b_\nu \, \mathrm{tr}\left[T_a T_b\right]$

Fermionic Coset

- Bosonic part is always a subgroup of the isometry supergroup:

$$g = exp\left(\sum_i x_i T_i + \sum_\alpha \theta_\alpha Q_\alpha\right) \longrightarrow exp\left(\sum_\alpha \theta_\alpha Q_\alpha\right)$$

- Few fermions: $g \longrightarrow \prod_\alpha exp\left(\theta_\alpha Q_\alpha\right)$

Simple Examples

$\frac{Osp(1|2)}{Sp(2)}$:

- Action:

$$S \sim \int_\Sigma \mathrm{d}^2 z \, (1 + \theta_1\theta_2)\, \varepsilon_{\alpha\beta} \partial\theta^\alpha \bar{\partial}\theta^\beta$$

- Isometry:

$$\theta_i \to \theta_i + (1 + \theta_1\theta_2)\, \epsilon_i \qquad \epsilon_i = \mathrm{cost}$$

How to perform T-duality?

$\frac{Osp(2|2)}{SO(2) \times Sp(2)}$:

- Action:

$$S \sim \int_\Sigma \left\{ \mathrm{d}\theta_1 \wedge *\mathrm{d}\theta_4 - \mathrm{d}\theta_2 \wedge *\mathrm{d}\theta_3 - 4\theta_3\theta_4 \mathrm{d}\theta_1 \wedge *\mathrm{d}\theta_2 \right\}$$

- Two translational isometries
- Equation of Motion: $4\theta_3\theta_4 * A_2 = *\mathrm{d}\theta_4 - \mathrm{d}\tilde{\theta}_2$

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- general coset model: $S = \int_\Sigma \mathrm{d}^2 z \sqrt{-\eta}\eta^{\mu\nu} \; V^a_\mu V^b_\nu \,\mathrm{tr}\,[T_a T_b]$
- isometries generated by Killing vectors $X^i \to X^i + \lambda^\Lambda K^i_\Lambda$
- transformation of the Vielbein: $V^A \to V^A + \mathrm{d}\lambda^\Lambda \, i_{K_\Lambda} V^A + \dots$
- gauging: $V^a \to V^a + A^a$

gauge fixing $\Rightarrow$ Solving $\quad \mathrm{d}\lambda^\Lambda = -V^a (i_{K_\Lambda} V^a)^{-1}$

Two conditions:

i existence of a minor: $\exists \quad M_I{}^s \equiv \left(i_{K_I} V^s\right) \quad \text{invertible}$

ii existence of a solution of $\mathrm{d}\left[V^s (M^{-1})_s{}^g\right] = 0$

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Non Linear Isometries

- Action for coset spaces models: depends on parametrization
- Non-linear isometry $\longrightarrow$ cumbersome procedure
- Fermionic coordinates: change of basis not always possible

Wayout: Plücker Relations

- Introduction of new coordinates and constraints
- Solving the constraints $\longrightarrow$ back to the original action
- New action: the isometries act linearly

$$\mathcal{L} \sim (1+\theta_1\theta_2)\,\varepsilon_{\alpha\beta}\partial\theta^\alpha\bar{\partial}\theta^\beta \longrightarrow \mathcal{L} \sim \varepsilon_{\alpha\beta}\partial\theta^\alpha\bar{\partial}\theta^\beta - \partial\phi\bar{\partial}\phi + \alpha\left(\phi^2-\theta^2-1\right)$$

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- Lagrangian

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$$\theta^\alpha \to \theta^\alpha - \lambda^\alpha\phi - \omega^\alpha{}_\beta\theta^\beta\ , \qquad \phi \to \phi - \lambda^\alpha\theta_\alpha$$

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- Collecting the gauge fields

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- Gauging: $\nabla\theta^\alpha = \partial\theta^\alpha - A^\alpha\phi - A^\alpha{}_\beta\theta^\beta\ , \qquad \nabla\phi = \partial\phi - A^\alpha\theta_\alpha$
- Collecting the gauge fields

$$\mathcal{L}_{gauging} \sim \mathcal{L}_0 + \left(h^\alpha + f^{\alpha\beta}A_\beta + g^{\alpha(\beta\gamma)}A_{\beta\gamma}\right)\bar{A}_\alpha + \left(l^{(\alpha\beta)} + m^{(\alpha\beta)\rho}A_\rho + n^{(\alpha\beta)(\rho\sigma)}A_{\rho\sigma}\right)\bar{A}_{\alpha\beta} + \bar{h}^\alpha A_\alpha + \bar{l}^{(\alpha\beta)}A_{\alpha\beta}$$

New Method: $\frac{OSp(1|2)}{Sp(2)}$

- Lagrangian

$$\mathcal{L}_0 \sim \varepsilon_{\alpha\beta}\partial\theta^\alpha\bar{\partial}\theta^\beta - \partial\phi\bar{\partial}\phi + \alpha\left(\phi^2 - \theta^2 - 1\right)$$

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- Gauge fixing $\oplus$ EoM for gauge fields $\to$ T-dual model

General Model: $\frac{OSp(n|m)}{SO(n) \times Sp(m)}$

- Supergroup representative: $L = \begin{pmatrix} \Lambda^i{}_j & \Theta^i{}_\alpha \\ \Theta^\alpha{}_j & \Phi^\alpha{}_\beta \end{pmatrix}$
  - $\Lambda_{(ij)}$ bosonic $SO(n)$ fields;
  - $\Phi_{[\alpha\beta]}$ antisymmetric $Sp(m)$ fields;
  - $\Theta_{i\alpha}$ fermionic fields.
- Vielbein: expand $L^{-1}\partial L$ over $\frac{\mathfrak{osp}(n|m)}{\mathfrak{so}(n) \times \mathfrak{sp}(m)}$ generators

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_P$$

- $\mathcal{L}_V = V^\alpha{}_i \delta^{ij} \varepsilon_{\alpha\beta} V^\beta{}_j + V^i{}_\alpha \delta_{ij} \varepsilon^{\alpha\beta} V^j{}_\beta$
- $\mathcal{L}_P = \alpha^{(ij)} \left(\Lambda_{ik} \delta^{kl} \Lambda_{lj} - \Theta_{i\alpha} \varepsilon^{\alpha\beta} \Theta_{j\beta} - \delta_{ij} \right) + \beta^{[\alpha\beta]} \left(\Phi_{\alpha\gamma} \varepsilon^{\gamma\delta} \Phi_{\delta\beta} - \Theta_{i\alpha} \delta^{ij} \Theta_{j\beta} - \varepsilon_{\alpha\beta} \right) + \gamma^{i\alpha} \left(\Lambda_{ik} \delta^{kl} \Theta_{k\alpha} + \Phi_{\alpha\beta} \varepsilon^{\beta\gamma} \Theta_{i\gamma} \right)$

Construction of T-dual Model!

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$\frac{OSp(n|m)}{SO(n) \times Sp(m)}$: Quantum Analysis

- Computation of 1- and 2-loop correction to self energy

Background Field Method:

- Group Representative: $g = g_0 e^{\lambda X}$
- background external field $B_\mu = g_0^{-1}\partial_\mu g$, quantum internal field X
- Vielbein: $V_\mu = B_\mu + \frac{\lambda^2}{2}\left[[B_\mu, X], X\right] + \lambda\partial_\mu X + \frac{\lambda^3}{3!}\left[[\partial_\mu X, X], X\right] + \dots$
- Action: $S = \frac{1}{2\pi\lambda^2}\int Str\,(V \cdot V) \longrightarrow$ Feynman Diagrams

Self Energy, both 1 and 2-loops $\propto (2 + m - n)$

- 1-loop computation for T-dual model $\longrightarrow$ same result!

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Results and Open Issues

Classical Analysis:

- Study of obstructions to Fermionic T-duality
- New method based on Plücker relations
- Construction of T-dual of $\dfrac{OSp(n|m)}{SO(n) \times Sp(m)}$

Quantum Analysis:

- BFM
- 1- and 2-loop Self Energy
- 1- and 2-loop Self Energy for T-dual model

Open Problems:

- T-duality for other models
- Analysis of WZW models
- Deep understanding of quantum T-duality

Contents

Fluid-Gravity Correspondence 1

- *AdS*/*CFT* correspondence

Type IIB string theory on asymptotically $AdS_5 \times S^5$

$\mathcal{N} = 4$ SYM on $S^3 \times R^1$ or $R^3 \times R^1$

- Effective Description: length scales $>>$ mean free path length

Einstein Equations with negative cosmological constant

Relativistic Fluid Dynamics

- Static Solution

Black Hole or Black Branes in *AdS*...

static perfect fluid configuration

- Perturbation

Evolving Black Brane

dissipative fluid flow

Fluid-Gravity Correspondence 2

Interesting Recent Results from Fluid-Gravity Correspondence:

Battacharyya, Hubeney, Minwalla, Rangamani – arXiv:0712.2456

- Perturbative procedure for Fluid-Gravity Correspondence
- Gravitational derivation of Navier-Stokes Equations

fluid dynamic $=$ long wavelength effective theory for strongly coupled QFT

- Explicit construction of Fluid Energy Momentum Tensor

new method to compute Transport Coefficients

- Sector of IIB SUGRA: pure gravitational dynamics in asymptotically *AdS*

- AdS_5 Black Brane in Poincaré patch:

$$2\mathrm{d}s_{BH} = \mathrm{d}v\,\mathrm{d}r - r^2 f(r)\,\mathrm{d}v^2 + r^2 \mathrm{d}\mathbf{x}^2 \qquad f(r) = 1 - \frac{1}{r^4}$$

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Basic Setup

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- Parameters functions of boundary coords $\longrightarrow$ not a solution of Einstein Eq.
- Perturbative Metric Expansion

$$G = G^0\left(\beta_i, b\right) + \varepsilon G^1\left(\beta_i, b\right) + O\left(\varepsilon^2\right)$$

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- Constraint Einstein Equation: (linearized) NS equations

$$\partial_v b^{(0)} = \frac{1}{3}\partial_i \beta_i^{(0)}\ , \qquad \partial_i b^{(0)} = \partial_v \beta_i^{(0)}$$

- Dynamic Einstein Equations: first order metric g^1
- Ultralocal metric up to first order $\rightarrow$ global solution to first order in derivatives
- Energy Momentum Tensor for Conformal fluid from Brown-York prescription

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Contribution from Black Hole Superpartners

- Pure Gravity $g_{\mu\nu} \longrightarrow$ Pure Supergravity $g_{\mu\nu}\,, \psi_\mu$:
- Supersymmetry transformation:

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Burrington, Liu, Sabra: hep-th/0412155

- AdS_5 without BH: no deformations:

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Killing Vectors

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- New metric term depending on fermionic bilinears $\{\lambda\,,\; l_i\}$

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- New metric term depending on fermionic bilinears $\{\lambda\,,\ l_i\}$

Killing Spinors

- Computation of the 8 AdS_5 Killing Spinors ϵ:

$$\delta_\epsilon \psi_\mu = \left(D_\mu(\omega) + \frac{1}{2}\Gamma_\mu \right) \epsilon = 0$$

- Notice that $\bar{\epsilon}\,\Gamma^\mu \epsilon\; \partial_\mu$ is the AdS_5 Killing vector
- Turning on the BH: $\delta_\epsilon \psi_\mu \neq 0$
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Results

- We reproduce the computation by Minwalla *et al.*

- 1^{st}–order metric with local parameters

$$g = g^{BH} + \varepsilon \left[\mathcal{L}_K g_{BH} \left(\beta_i, b, \alpha_i, a \right) + \delta_\epsilon^2 g_{BH} \left(l_i, \lambda \right) + g^1 \right]$$

- Imposing Einstein Equations in large r limit $\longrightarrow$ 4 New Constraints eq's:

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- Correction to NS equations!

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Road to Supersymmetric Fluid Dynamics 1

Interpretation of modifyed NS? $\longrightarrow$ we need a Supersymmetric Fluid Dynamics

P.A. Grassi, AM, L. Sommovigo – arXiv:1107.2780

- Idea: supersymmetrize an appropriate bosonic Lagrangian
- Bosonic model using Clebsh Parametrization:

$$\mathcal{L} = \sqrt{-g}\left(j^{\mu}\left(\partial_{\mu}a + \alpha\partial_{\mu}\beta\right) + f\left(j^{2}\right)\right)$$

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- Other EoM's $\oplus$ manipulations $\longrightarrow$ NS equations (no dissipation terms)

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Supersymmetrization

- $j_\mu \rightarrow$ linear superfield J satisfying $\bar{D}DJ = 0$
- $\partial_\mu a + \alpha \partial_\mu \beta \rightarrow$ real superfield A in Wess-Zumino gauge
- introduction of $\mathcal{J}^\mu \propto \left(\bar{D}\gamma_5\gamma_\mu D\right) J$

$$S = \int \mathrm{d}^4 x \int \mathrm{d}^4\theta \left(-J\,A + F\left(\mathcal{J}_\mu \mathcal{J}^\mu\right) J^2\right)$$

Results

- Construction of Supersymmetric Lagrangian
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- Supersymmetric version of Clebsh parametrization (not seen here)

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Road to Supersymmetric Fluid Dynamics 2

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Results and Open Issues

Fluid-Gravity Correspondence:

- Application of perturbative procedure *a lá* Minwalla to Black Hole Superpartners
- Derivation of NS equations modified by fermionic dof's

Models for Supersymmetric Fluid Dynamics:

- Construction of suitable supersymmetric lagrangian
- Analysis of different sectors

In preparation:

- Computation of boundary Energy Momentum Tensor
- Derivation of modified NS from supersymmetric Lagrangian
- Supergravity Extension

Contents

List of Publications

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Thank You!

Conclusions

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