Power suppressed operators in inclusive semileptonic B decays

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Overview

Semileptonic B decays provide the most precise determination of the CKM matrix elements $|V_{cb}|$ and $|V_{ub}|$.

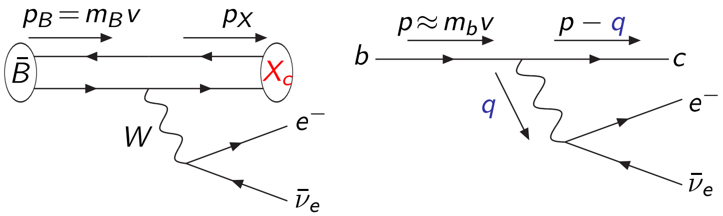
With the coming in a few years of new high luminosity B factories, theoretical uncertainties should be reduced wherever possible:

there still is plenty of perturbative corrections to be computed.

Our calculation and our understanding of such phenomena are founded on two devices in particular:

- OPE - the Operator Product Expansion
- HQET - the Heavy Quark Effective Theory

Inclusive Semileptonic $\bar{B}$ Decay into X_c



here is the process under exam, since we are summing over X_c the W invariant mass q^2 is a variable as well as E_ν and E_e.
The triple differential decay distribution will be:

$$\frac{d\Gamma}{dq^2 dE_\nu dE_e} = \frac{1}{4} \sum_{X_c} \sum_{spin} \frac{|<X_c e\bar{\nu}|H_w|\bar{B}>|^2}{2m_B} \delta^4(p_B - p_e - p_\nu - p_X)$$

where the interaction is described by the weak hamiltonian H_w:

$$H_w = \frac{4G_F}{\sqrt{2}} V_{cb} \; J_L^\mu \; \bar{e}\gamma_\mu P_L \nu_e \qquad J_L^\mu = \bar{c}\gamma^\mu P_L b \qquad P_L = \frac{1}{2}(1 - \gamma_5)$$

Leptonic and Hadronic Tensors

Leptons don't have strong interactions, so we can further decompose the differential decay rate as follows:

$$\frac{d\Gamma}{dq^2 dE_\nu dE_e} = 2G_F^2 |V_{cb}|^2 W_{\alpha\beta} L^{\alpha\beta}$$

the weak matrix element has thus been factored into two parts:

- the leptonic tensor $L^{\alpha\beta}$

$$L^{\alpha\beta} = 2\left(p_e^\alpha p_\nu^\beta + p_\nu^\alpha p_e^\beta - p_e \cdot p_\nu g^{\alpha\beta} - i\epsilon^{\rho\beta\sigma\alpha} p_{e\,\rho} p_{\nu\,\sigma}\right)$$

- the hadronic tensor $W^{\alpha\beta}$

$$W^{\alpha\beta} = \frac{(2\pi)^3}{2m_B} \sum_{X_c} \delta^4(p_b - q - p_X) < \bar{B}|J_L^{\dagger\alpha}|X_c >< X_c|J_L^\beta|\bar{B} >$$

Structure Functions W_i

The most general form of the tensor $W^{\alpha\beta}$ can be written as

$$W^{\alpha\beta} = -g^{\alpha\beta} W_1 + v^\alpha v^\beta W_2 - i\epsilon^{\alpha\beta\mu\nu} v_\mu q_\nu W_3 + q^\alpha q^\beta W_4 + (v^\alpha q^\beta + q^\alpha v^\beta) W_5$$

W_4 and W_5 do not contribute to the final result because

$$q_\alpha L^{\alpha\beta} = q_\beta L^{\alpha\beta} = 0$$

in the end only W_1, W_2 and W_3 survive

$$\frac{d\Gamma}{dq^2 dE_\nu dE_e} = \frac{G_F^2 |V_{cb}|^2}{2\pi^3} \left[q^2 W_1 + \left(2E_e E_\nu - \frac{q^2}{2} \right) W_2 + q^2 (E_e - E_\nu) W_3 \right]$$

with the use of projectors we can just compute the W_i structure functions, purely scalar objects, rather than the whole tensor $W^{\alpha\beta}$.

Time Ordered Product of Currents

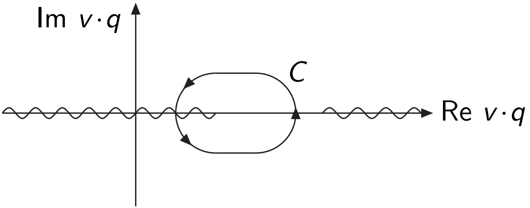
The hadronic tensor $W^{\alpha\beta}$ can be related to the discontinuity of a T product of currents across a cut. So if we define:

$$T^{\alpha\beta} = -i \int d^4x e^{-iq\cdot x} \frac{1}{2m_B} < \bar{B}| T \left[J_L^{\dagger\alpha}(x) J_L^{\beta}(0) \right] |\bar{B} >$$

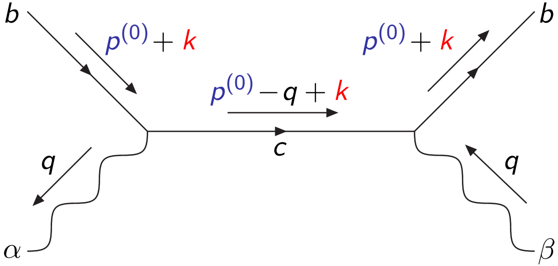
then we obtain the relation

$$W_i = -\frac{1}{\pi} \mathrm{Im}\, T_i$$

these are the cuts of some T_i at fixed q^2, C is the integral contour



Kinematics



this diagram represents $T^{\alpha\beta}$, if we set the b quark on-shell

$$p_\mu = p_\mu^{(0)} = m_b v_\mu \qquad v^2 = 1$$

that would reproduce the decay of a free quark.
Instead we assume that momentum p_μ is a little off-shell:

$$p_\mu = p_\mu^{(0)} + k_\mu \qquad < \frac{k \cdot v}{q \cdot v} >= O\left(\frac{\Lambda_{QCD}}{m_b}\right)$$

thus taking into account the interactions inside the $\bar{B}$ meson.

Operator Product Expansion

We are considering a time-ordered product of currents, so we can employ the Operator Product Expansion technique to express it as a series of local operators:

$$T\left[J_L^{\dagger\alpha}(x)J_L^{\beta}(0)\right] = \sum_i C_i O_i(x)$$

the off-shell component of the momentum k_μ has an immediate interpretation in terms of the QCD covariant derivative D_μ

$$\bar{b}\, k_\mu b \;\rightarrow\; \bar{b}(iD_\mu - m_b v_\mu)b; \qquad D_\mu = \partial_\mu + iG_\mu^a T^a$$

so we can Taylor expand our whole expression in k_μ and expect a result in the form of a nonperturbative series in m_b

$$T^i = \sum_{n\geq 3}\sum_j \left(\frac{1}{m_b}\right)^{n-3} c_{(n)}^{ij} < \bar{B}|O_j^{i(n)}|\bar{B} >$$

Zeroth order - $k = 0$

At the lowest order of the expansion we just set:

$$k_\mu = 0$$

at this level only one operator arises,

$$O_b = \bar{b}\gamma_\mu b$$

which is readily evaluated between $\bar{B}$ states:

$$< O_b >= v_\mu \qquad < O >= \frac{1}{2m_B} < \bar{B}|O|\bar{B} >$$

the final result of course reproduces the decay of a free b quark.

Such degree of approximation is simple enough to be easily described by QCD alone, on the other hand contributions of higher order might require a more suitable formalism.

Heavy Quark Effective Theory

HQET is an effective field theory tailored to deal with the dynamics of hadrons containing a single heavy quark.

Its description is valid when dealing with momenta much smaller than the heavy quark mass - for us that's m_b.

The effective lagrangian is conveniently formulated in terms of velocity-dependent fields $Q_v(x)$:

$$Q_v(x) = e^{im_b v\cdot x}\,\frac{1+\not{v}}{2}Q(x); \qquad \bar{Q}_v(x) = e^{im_b v\cdot x}\,\frac{1-\not{v}}{2}Q(x);$$

so that the original QCD quark field $Q(x)$ can be written as

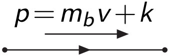
$$Q(x) = e^{-im_b v\cdot x}\left[Q_v(x) + \bar{Q}_v(x)\right]$$

HQET - Vertices and Propagators

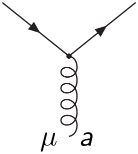
Substituting the effective fields Q_v and $\bar{Q}_v$ into the QCD lagrangian we obtain a series in inverse powers of m_b:

$$L_{eff} = \bar{Q}_v \left(iv \cdot D - \frac{1}{2m_b} \not{D} \not{D} \right) Q_v + O\left(\frac{1}{m_b^2} \right)$$

at the lowest order the effect on the theory is the same as taking the $m_b \to \infty$ limit of the QCD Feynman rules, assuming the heavy quark to be a little off-shell:



$$i \frac{\not{p} + m_b}{p^2 - m_b^2 + i\epsilon} \quad \to \quad i \frac{1 + \not{v}}{2v \cdot k + i\epsilon}$$



$$-ig_S T^a \gamma_\mu \quad \to \quad -ig_S T^a v_\mu$$

HQET - Operators and Parameters

We won't compute HQET diagrams, but the effective theory can tells us what operators to look for and how to parametrize them:

- only one operator at dimension-four

$$< \bar{b}_v iD_\mu b_v >= O\left(\frac{1}{m_b}\right) \qquad \bar{b}_v(iv\cdot D)b_v = \frac{1}{2m_b}\bar{b}_v \not{D}\not{D}\, b_v$$

- the kinetic operator at dimension-five

$$< \bar{b}_v iD_{(\mu} iD_{\nu)} b_v >= \frac{\lambda_1}{3}(g_{\mu\nu} - v_\mu v_\nu) + O\left(\frac{1}{m_b}\right)$$

- the choromagnetic operator still at dimension-five

$$< \bar{b}_v iD_{[\mu} iD_{\alpha]} \sigma^{\mu\beta} b_v >= -2\lambda_2(g_\alpha^\beta - v_\alpha v^\beta) + O\left(\frac{1}{m_b}\right)$$

- more operators at dimension-six and beyond

Calculation of λ_1

We're all set to determine the complete coefficient of λ_1.

We expand all propagators to the second order in k and replace the QCD b-quark fields according to HQET

$$b(x) = e^{-im_b v\cdot x}\left(1 + \frac{i\not{D}}{2m_b}\right) b_v(x)$$

so that we may recognize the operators we expect

$$\bar{b}(iD_\mu - m_b v_\mu)b \;\rightarrow\; \bar{b}_v iD_\mu b_v$$

then we evaluate all relevant matrix elements and get the full result

$$< \bar{b}_v iD_\mu \gamma_\nu b_v > = -\frac{\lambda_1}{2m_b} v_\mu v_\nu$$

$$< \bar{b}_v iD_{(\mu} iD_{\nu)} \gamma_\rho b_v > = \frac{\lambda_1}{3}(g_{\mu\nu} - v_\mu v_\nu) v_\rho$$

Looking for λ_2

The coefficient of λ_2 is not as straightforward.

The contribution from $\bar{b}_v iD_\mu b_v$ does not pose any problem, its matrix element follows from the HQET equation of motion:

$$< \bar{b}_v iD_\mu \gamma_\nu b_v >= -\frac{3\lambda_2}{2m_b} v_\mu v_\nu$$

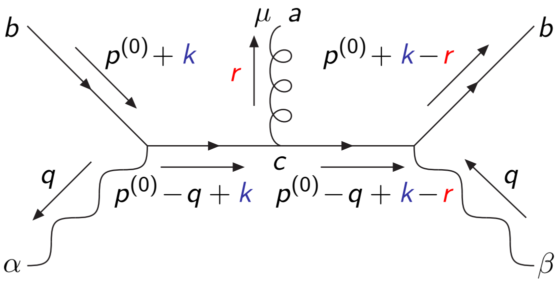
What really hampers us is to find a structure like $iD_{[\mu} iD_{\nu]}$ amidst an expression that is symmetric in $k_\mu k_\nu$ by construction.

We must devise some other way to extract the contribution of the chromomagnetic operator to $W^{\alpha\beta}$.

Soft External Gluon

If we add the emission of a soft gluon to our previous diagram, the process is still perfectly legitimate inside a $\bar{B}$ meson.

The gluon has virtuality r_μ, comparable to k_μ



the idea is to try and expand unitely in k_μ and r_μ.

Gluon Fields inside the Operators

Each operator inside the expansion includes different kinds of components that we can see in different kind of processes.

Consider for istance the kinetic operator:

$$iD_{(\mu}iD_{\nu)} = i\partial_\mu i\partial_\nu - G^a_{(\mu}T^a i\partial_{\nu)} - \frac{i}{2}\partial_{(\mu}\left[G^a_{\nu)}T^a\right] + \frac{1}{2}G^a_\mu G^b_\nu\left(T^aT^b + T^bT^a\right)$$

the terms without a gluon field are to be seen in the old diagram with no external gluon, while those containing G_μ can be found in the new diagram emitting a soft gluon.

On the other hand, all terms inside the chromomagnetic operator have at least one gluon field, so we need that new diagram

$$iD_{[\mu}iD_{\nu]}\sigma^{\mu\alpha} = \left[i\partial_\nu\left(G^a_\mu\right) - i\partial_\mu\left(G^a_\nu\right)\right]T^a\sigma^{\mu\alpha} + G^a_\mu G^b_\nu\left(T^bT^a - T^aT^b\right)\sigma^{\mu\alpha}$$

At last, λ_2

What we have to do is to find the imprint of each operator in our new expression. The basic relations we need are:

$$i\partial_\mu \;\rightarrow\; k_\mu \qquad G^a_\mu T^a \;\rightarrow\; t^a_\mu T^a \qquad i\partial_\mu \left[G^a_\nu T^a\right] \;\rightarrow\; -r_\mu t^a_\nu T^a$$

we can calculate again the complete coefficient of λ_1 and the result we find is the same as before:

$$iD_\mu \;\rightarrow\; -t^a_\mu T^a$$

$$iD_{(\mu}iD_{\nu)} \;\rightarrow\; -\left(k_\mu t^a_\nu + k_\nu t^a_\mu\right) T^a + \frac{1}{2}\left(r_\mu t^a_\nu + r_\nu t^a_\mu\right) T^a$$

then we are finally able to get the last contribution to λ_2

$$iD_{[\mu}iD_{\nu]}\sigma^{\mu\alpha} \;\rightarrow\; \left(r_\nu t^a_\mu - r_\mu t^a_\nu\right) T^a \sigma^{\mu\alpha}$$

Phase Space

Once we have all contributions for λ_1 and λ_2 we can perform the phase space integration and obtain physically relevant moments.

$$< M(n,m,l) > = \int d\Phi \, \frac{d\Gamma}{dq^2 dE_\nu dE_e} \, E_e^n E_X^m u^l$$

where $u = (p^{(0)} - q)^2 - m_c^2$ and $E_X = m_b^2 - m_b q \cdot v$.
The phase space integration itself can be parametrized as follows:

$$\int d\Phi = \int_{\Lambda_{cut}}^{\frac{m_b^2 - m_c^2}{2}} dE_e \int_0^{\frac{2E_e}{2E_e - m_b^2}(2E_e - m_b^2 + m_c^2)} dq^2 \int_0^{m_b^2 - 2E_e - m_c^2 + q^2\left(1 - \frac{m_b^2}{2E_e}\right)} du$$

including a lower cut Λ_{cut} on the lepton energy. The total rate is

$$\Gamma_T = \Gamma_0 \left(1 + \frac{\lambda_1}{2m_b^2} - 5.82 \frac{\lambda_2}{m_b^2}\right)$$

with $\Gamma_0 = \frac{G_F^2 m_b^5}{192\pi^3}\left(1 - 8\rho + 8\rho^3 - \rho^4 - 12\rho^2 \ln\rho\right)$ and $\rho = \frac{m_c^2}{m_b^2}$.

Perturbative Corrections

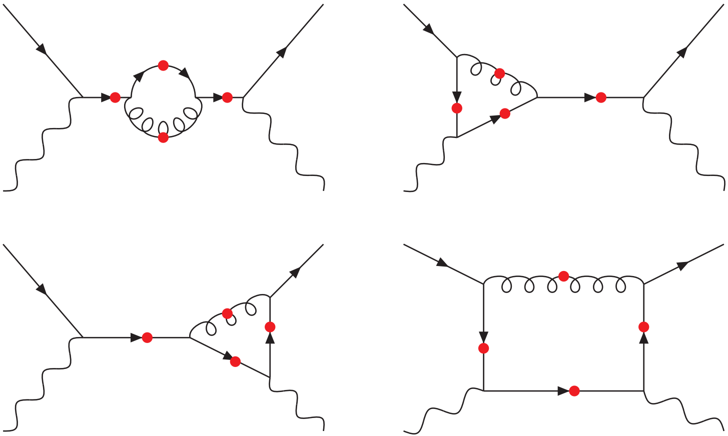
The tree level results have long been known, on the contrary NLO corrections to the operators coefficients were computed much more recently and only for the free decay and the kinetic operator.

The technical devices employed in the 1-loop calculation are nothing unheard of, only a few aspects deserve mentioning:

- the benefits of working with scalar objects
  - separation betwenn k_μ, r_μ and loop momentum l_μ
  - reduction to master integrals with IBP identities
- cancellation of divergencies
  - renormalization
  - matching

Relevant Diagrams

These are all the diagrams at order $O(\alpha_S)$



the red dots mark all possible insertions of an external gluon

IBP - Integration By Parts

The IBP identities are derived by differentiating many loop integrals with respect to the loop momentum l_μ, thus creating a large set of interloped constrains that can be solved

$$\frac{\partial}{\partial l^\mu} \int d^4 l \, \frac{l \cdot q}{(l-p)^2 - m^2} = 0 \rightarrow \int d^4 l \, \frac{\partial}{\partial l^\mu} \frac{l \cdot q}{(l-p)^2 - m^2} = 0$$

so for each scalar loop integral we have to deal with

$$I_{abc} = \int d^4 l \, [l^2]^{-a} [(l + p^{(0)})^2 - m_b^2]^{-b} [(l - q + p^{(0)})^2 - m_c^2]^{-c}$$

we can replace a linear combination of master integrals:

$$I_{abc} = C^{001}_{abc} I_{001} + C^{010}_{abc} I_{010} + C^{011}_{acb} I_{011} + C^{101}_{abc} I_{101} + C^{111}_{abc} I_{111}$$

Renormalization

We use dimensional regularization to manipulate divergencies

$$d = 4 - 2\epsilon$$

UV poles are removed by renormalization. The counterterms are:

- Wave Function Renormalization on the external legs

$$\delta Z_b^{OS} = -3C_F\left(\frac{1}{\epsilon} + \frac{4}{3}\right)\frac{\alpha_S}{4\pi}$$

- mass renormalization for the internal charm propagator

$$\delta m_c = -4m_c C_F\left(\frac{1}{\epsilon} + \frac{3}{4} - \frac{3}{4}\ln\frac{m_c}{m_b}\right)\frac{\alpha_S}{4\pi}$$

- renormalization of the kinetic operator

$$\delta Z_{kin}^{\mu\nu\alpha\beta} = -2C_F\frac{1}{\epsilon}(g^{\mu\nu} - 2v^\mu v^\nu)v^\alpha v^\beta\frac{\alpha_S}{4\pi}$$

- renormalization of the chromomagnetic operator

$$\delta Z_{chromo}^{\mu\nu\alpha\beta} = C_A\frac{1}{\epsilon}(g^{\mu\alpha} - v^\mu v^\alpha)g^{\nu\beta}\frac{\alpha_S}{4\pi}$$

Matching

What we are attempting to do is to match the tensor $T^{\alpha\beta}$ with a nonperturbative series of HQET and QCD operators

$$T_i = \sum_j C_i^j O_i$$

the rhs perturbative corrections are inside the C_i^j coefficients.

Getting rid of UV poles required renormalization: WFR and mass counterterm on the left, operator renormalization on the right.

There still remain IR divergencies, but QCD and HQET reproduce by construction the same infrared behaviour, so in the end all poles cancel between the right-hand side and the left-hand side.

Was it Worthwhile?

Once the numerical integration has been performed including the perturbative corrections, we can argue whether these new contributions are sizable or not:

- inside the total rate, the coefficient of λ_2 increases by $+7\%$

$$\Gamma_T = \Gamma_0 \left[\left(1 - 1.78\frac{\alpha_S}{\pi}\right)\left(1 + \frac{\lambda_1}{2m_b^2}\right) - \left(1.94 + 2.42\frac{\alpha_S}{\pi}\right)\frac{3\lambda_2}{m_b^2} \right]$$

- $+28\%$ in the case of the mean lepton energy

$$< E_e >= 1.41\mathrm{GeV}\left[\left(1 - 0.02\frac{\alpha_S}{\pi}\right)\left(1 - \frac{\lambda_1}{2m_b^2}\right) - \left(1.19 + 4.20\frac{\alpha_S}{\pi}\right)\frac{3\lambda_2}{m_b^2}\right]$$

- an increase of $+23\%$ for the variance of the lepton energy

$$l_2 = 0.18\mathrm{GeV}^2\left[1 - 0.2\frac{\alpha_S}{\pi} - \left(4.9 - 0.4\frac{\alpha_S}{\pi}\right)\frac{\lambda_1}{m_b^2} - \left(2.9 + 8.4\frac{\alpha_S}{\pi}\right)\frac{3\lambda_2}{m_b^2}\right]$$

Still to be done ...

The work is far from finished:

- reliable numerical integration with the E_l cut
- separation of V and A components, right-handed currents
- $O(\frac{1}{m_b^3})$ corrections at NLO