Aspects of integrability in classical and quantum field theories

Second year seminar

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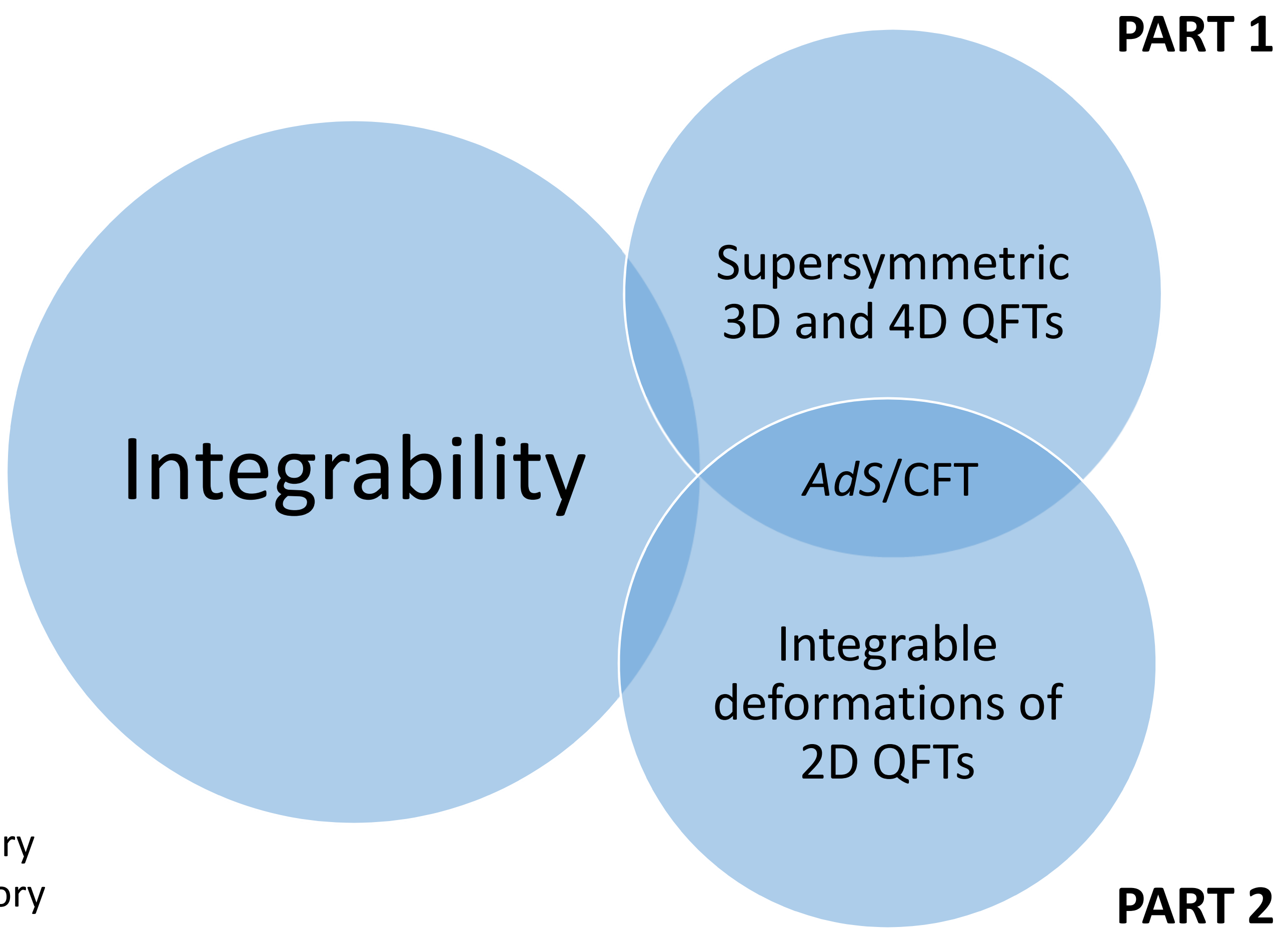
Plan of the talk



Vocabulary :
QFT = Quantum Field Theory
CFT = Conformal Field Theory
AdS = anti-de Sitter

Integrability in a nutshell

INTEGRABILITY ≈ EXACT SOLVABILITY

Physical observables are found analytically or by quadrature for any value of the parameters.

→ completely non-perturbative approach (no *Feynman diagrams*!)

Necessary conditions : infinite number of conservation's laws (in field theories).

Drawback : *few* systems are integrable! Usually *low dimensional* systems $(\mathrm{D} \leq 2)$

Examples : harmonic oscillator, Korteweg-deVries (KdV), sine-Gordon, Heisenberg spin chain,…

(hydrodynamics) (Kondo effect) (ferromagnets)

However…

- several applications to low dimensional physics;
- studying simpler (*integrable*) models allows to better understand more sophisticated ones.

PART 1

Supersymmetric 3D and 4D QFTs

Integrability in 3D and 4D QFTs

$\mathcal{N} = 4$ super Yang-Mills (SYM) in $\mathrm{D} = 4 \longrightarrow$ *Quantum Chromodinamic* (QCD).

$\mathcal{N} = 6$ super Chern-Simons (ABJM) in $\mathrm{D} = 3 \longrightarrow$ *Fractional Quantum Hall effect* (FQHE)

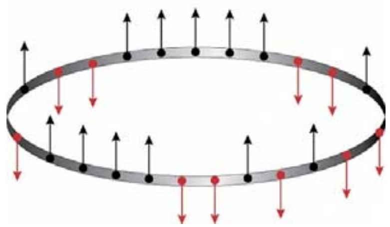
(supersymmetry + conformal symmetry)

- Why studying them? $\longrightarrow \begin{cases} \text{1) to get insights about very complicated } \textit{physical theories}; \\ \text{2) to study the } \textit{gauge}|\textit{gravity correspondence AdS}/\text{CFT}; \end{cases}$

- What are the observables? $\longrightarrow \begin{cases} \text{1) conformal dimensions } \Delta(h) \\ \text{2) structure constants } \{c_{ijk}(h)\} \end{cases}$

- How does integrability emerge? $\longrightarrow$ connection with 2D integrable systems (spin chains)

$$\Delta(h) = \Delta_0 + h^2 E_1 + h^4 E_2 + \cdots$$



E_n is the energy of a spin chain with interaction between n neighbouring sites.

Bombardelli, Cavaglià, Conti, Tateo, 10.1007/JHEP04(2018)117

Conformal dimensions in ABJM

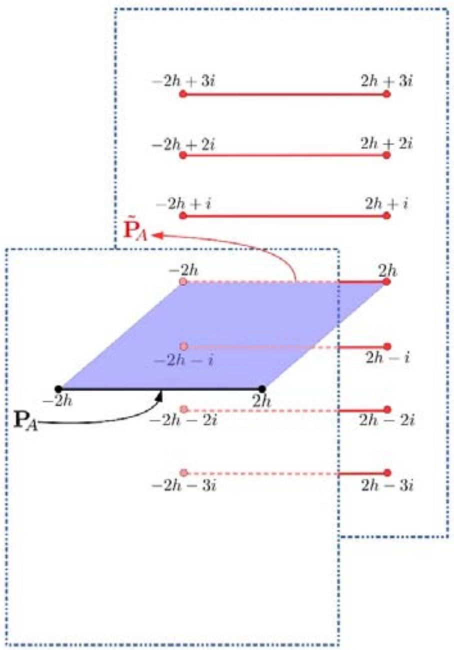
Quantum Spectral Curve (QSC)

Finite set of functional equations involving $6+6$ multi-valued functions, $\{\mathbf{P}_A(u)\}_{A=1,\ldots,6}$ and $\{\mathbf{Q}_I(u)\}_{I=1,\ldots,6}$

Riemann-Hilbert problem

INPUT: Large-u behavior of $\mathbf{P}_A(u)$ and $\mathbf{Q}_I(u)$ → QSC → OUTPUT: $\mathbf{P}_A(u)$ and $\mathbf{Q}_I(u)$ in the whole complex u-plane

u-plane



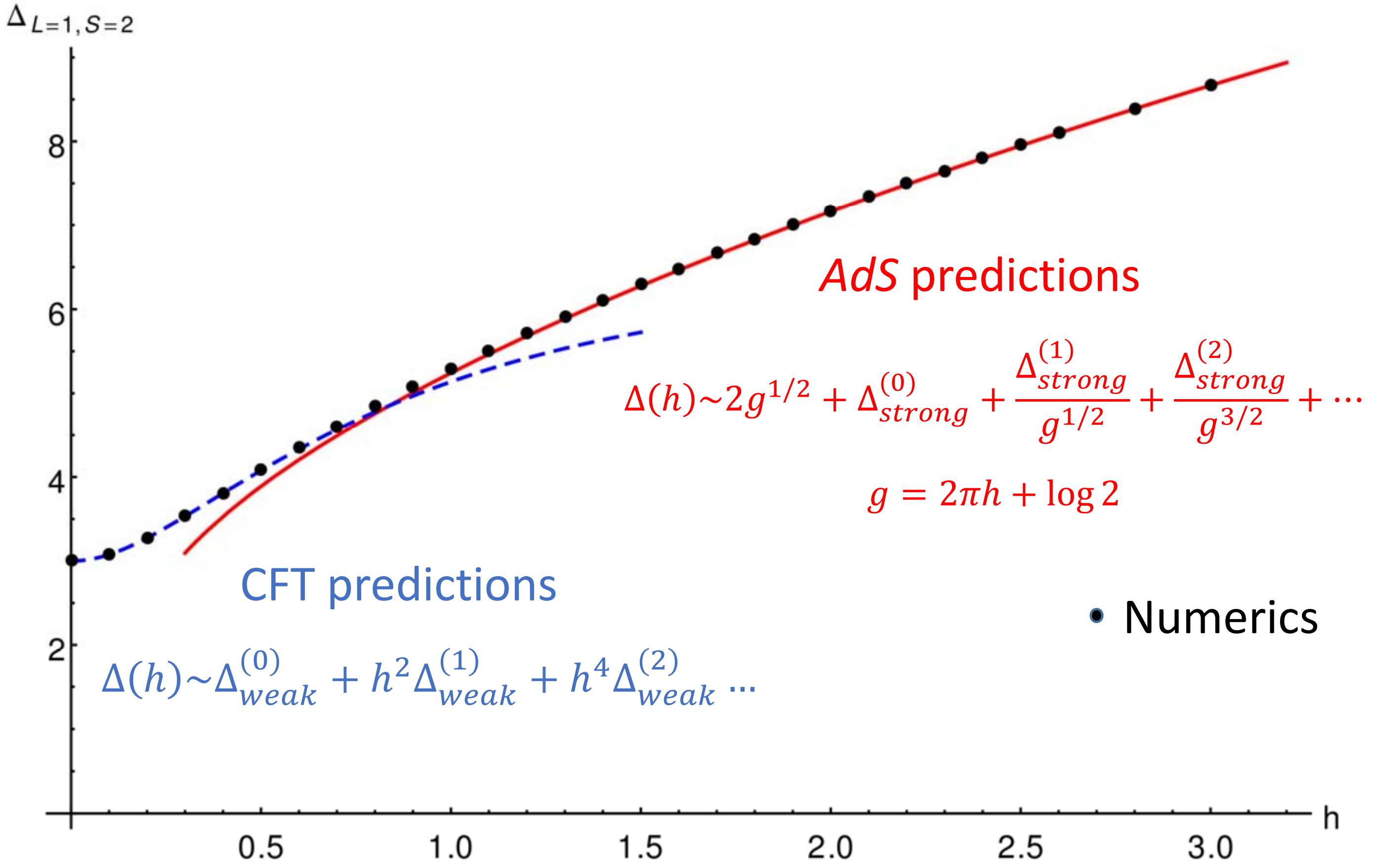
The conformal dimension $\Delta(h)$ is hidden in the asymptotics!

Idea : fix $\Delta(h)$ imposing the QSC equations

Bombardelli, Cavaglià, Conti, Tateo, [10.1007/JHEP04(2018)117](https://doi.org/10.1007/JHEP04(2018)117)

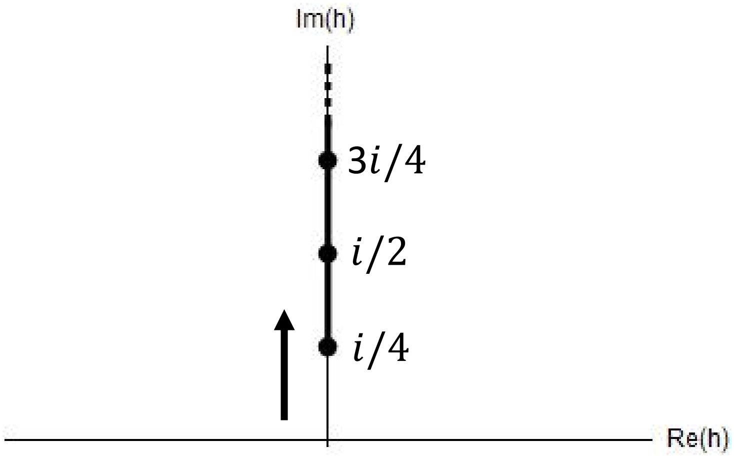
Numerical results

- Method to compute conformal dimensions of any operator at any value of the coupling constant h with *high precision*;
- Confirmation of the validity of the *AdS*/CFT correspondence.



Bombardelli, Cavaglià, Conti, Tateo (to appear)

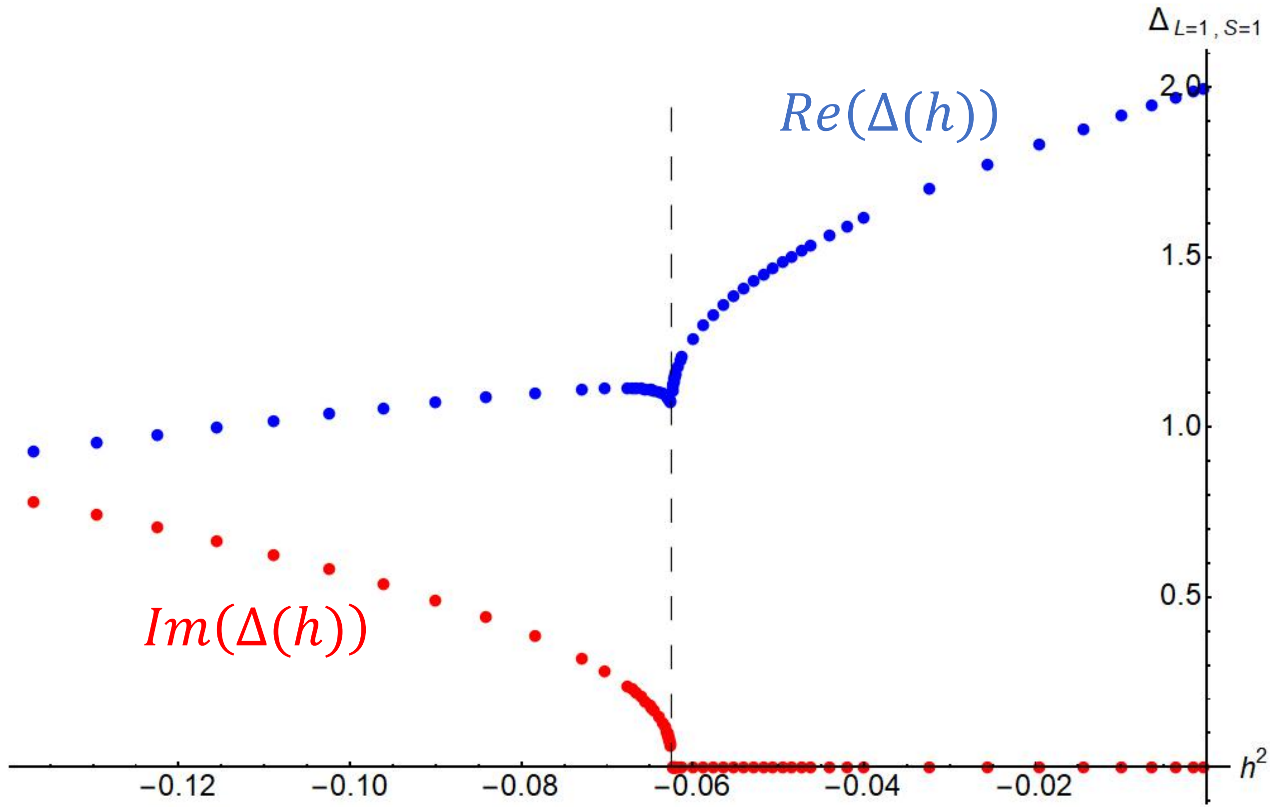
Analytic structure of $\Delta(h)$



- Motivation : phase transition $AdS \leftrightarrow dS$ when $h \to i\,h$ (Polyakov conjecture).

- study the critical exponent of $\Delta(h)$ around branch point singularities in the complex h-plane. Hinted presence of a logarithmic term

$$\Delta(h) \sim \alpha_0 + \left[\alpha_1 + \beta_1 \log\left(16h^2 - 1\right)\right]\left(16h^2 - 1\right)^{1/4}$$



PART 2

Integrable deformations of 2D QFTs

Integrable deformations of 2D QFTs

Generic 2D QFT with finite-size (R) spectrum $E_n(R,0)$

+ CDD factor

→ evolution equation for the finite-size spectrum $E_n(R,\tau)$

S-matrix : S_{ij}^{kl}

$S_{ij}^{kl} \longrightarrow S_{ij}^{kl} e^{\mathbb{i}\delta_{ij}(\tau)}$

Burgers equation : $\frac{\partial}{\partial\tau} E_n(R,\tau) = \frac{1}{2}\frac{\partial}{\partial R}\left(E_n^2(R,\tau) - P_n^2(R)\right)$

Solution : $E^2(R,\tau) - P^2(R) = E^2(\mathcal{R}_0, 0) - P^2(\mathcal{R}_0)$, where $\mathcal{R}_0^2 = (R + \tau\, E(R,\tau))^2 - (\tau\, P(R))^2$

$E(R,\tau)$ is uniquely determined from the initial condition $E(R,0)$ (integrable deformation)

Example : starting from the spectrum of a CFT , $E(R,0) = \frac{A}{R}$

$$E(R,\tau) = \frac{R}{2\tau}\left(-1 + \sqrt{1 + \frac{4\tau}{R^2}A + \frac{4\tau^2}{R^2}P^2(R)}\right)$$

Nambu-Goto energy levels (quantum gravity theory)

$\mathbf{T\bar{T}}$-deformation

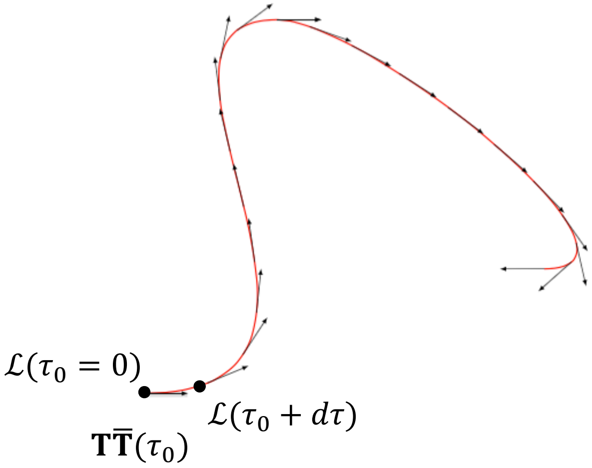
What is the generator of this deformation?

$\mathbf{T\bar{T}}$ operator : $\mathbf{T\bar{T}} \equiv T\bar{T} - \Theta^2$

Factorization property : $\langle n|\mathbf{T\bar{T}}|n\rangle = \langle n|T|n\rangle\langle n|\bar{T}|n\rangle - \langle n|\Theta|n\rangle\langle n|\Theta|n\rangle$

Burgers equation implies the flow equation :

$$\frac{\partial}{\partial\tau}\mathcal{L} = \mathbf{T\bar{T}}$$



Example : starting from N free bosons $\mathcal{L}^{(0)} = g^{\mu\nu}\partial_\mu\phi_i\partial_\nu\phi^i$, $\mu,\nu = 1{,}2$, $i = 1, \dots, N$

$$\mathcal{L}(\tau) = \frac{1}{2\tau}\left(-\sqrt{\det[g_{\mu\nu}]} + \sqrt{\det[g_{\mu\nu} + \tau\, h_{\mu\nu}]}\right) \quad , \quad h_{\mu\nu} = \partial_\mu\phi_i\partial_\nu\phi^i$$

Nambu-Goto Lagrangian in static gauge

Conti, Iannella, Negro, Tateo, arXiv:1806.11515 **[hep-th]**

Conti, Negro, Tateo, arXiv:1809.09593 **[hep-th]**

Geometric interpretation

Extension to a generic interacting bosonic theory : $\mathcal{L}_V^{(0)} = g^{\mu\nu}\partial_\mu\phi_i\partial_\nu\phi^i + V(\vec{\phi}) \quad , \quad \vec{\phi} = (\phi_1, \dots, \phi_N)$

$$\mathcal{L}_V(\tau) = \frac{V}{1-\tau V} + \frac{1}{2\bar{\tau}}\left(-\sqrt{\det[g_{\mu\nu}]} + \sqrt{\det[g_{\mu\nu} + \bar{\tau}\, h_{\mu\nu}]}\right) \quad , \quad h_{\mu\nu} = \partial_\mu\phi_i\partial_\nu\phi^i \quad , \quad \bar{\tau} = \tau(1-\tau V)$$

the equations of motion (EoMs) associated to $\mathcal{L}_V(\tau)$ are really complicated!

There exists a <u>field-dependent change of variables</u> $(x^1, x^2) \to (y^1(x^1,x^2), y^2(x^1,x^2))$ such that

EoMs of $\mathcal{L}_V(\tau)$ ⟺ EoMs of $\mathcal{L}_V^{(0)}$

$$\phi(x^1, x^2 |\, \tau) = \phi(y^1(x^1,x^2), y^2(x^1,x^2) |\, \tau = 0)$$

$\mathbf{T\overline{T}}$-deformed solutions on flat spacetime

$g_{\mu\nu} = \eta_{\mu\nu}$

undeformed solutions on curved spacetime

$g'_{\mu\nu} = \eta_{\mu\nu} - \tau\, \epsilon_{\mu\rho}\epsilon^{\sigma}{}_{\nu}\left(2T + \tau\, T^2\right)^{\rho}{}_{\sigma}$

Jackiw-Teitelboim (JT) topological gravity in 2D

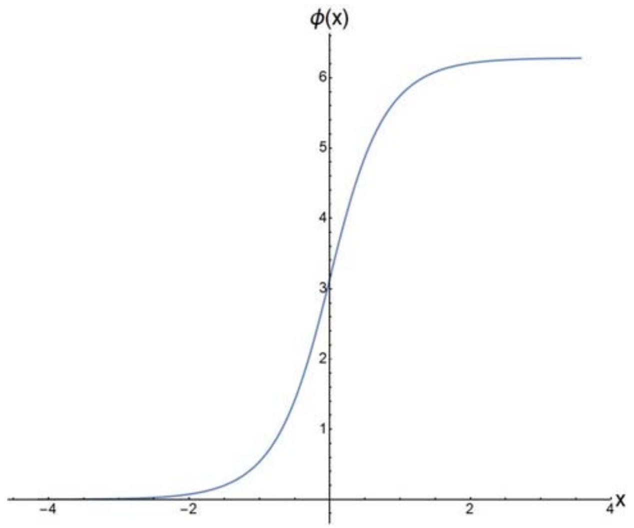
Study case : sine-Gordon model

sine-Gordon (sG) model is a 2D integrable theory described by the Lagrangian

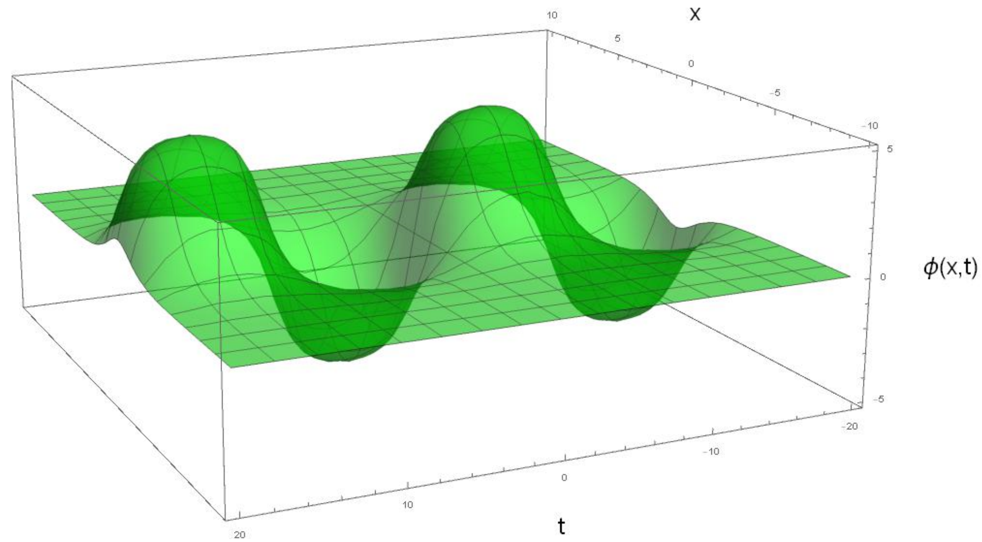
$$\mathcal{L}_{sG} = (\partial_x \phi)^2 - (\partial_t \phi)^2 + V(\phi) \quad , \quad V(\phi) = 4\left(\sin\frac{\phi}{2}\right)^2$$

solutions of sG are well known! ⟶ solitons

stationary 1-kink solution



stationary breather solution



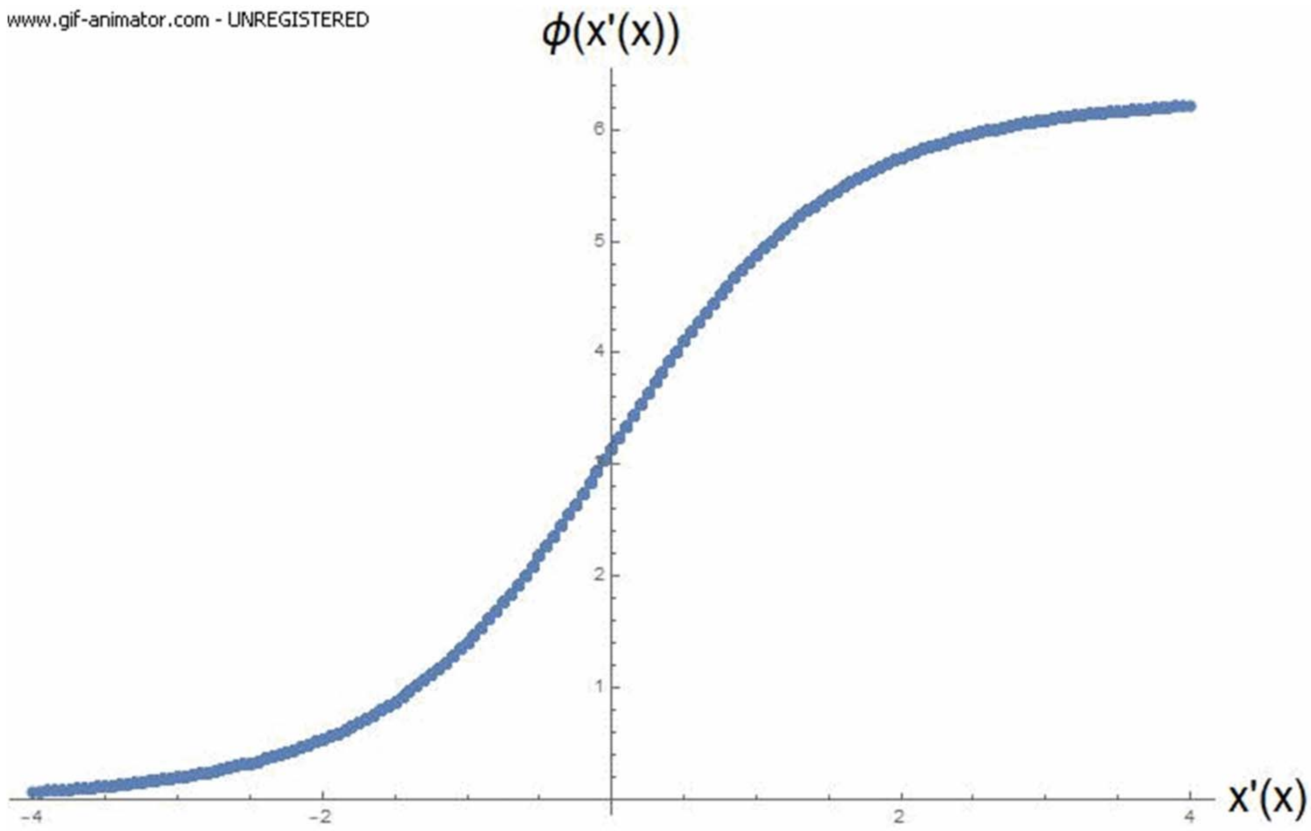
Deformed 1-kink solution

shock-wave singularity at $\tau = \frac{1}{8}$

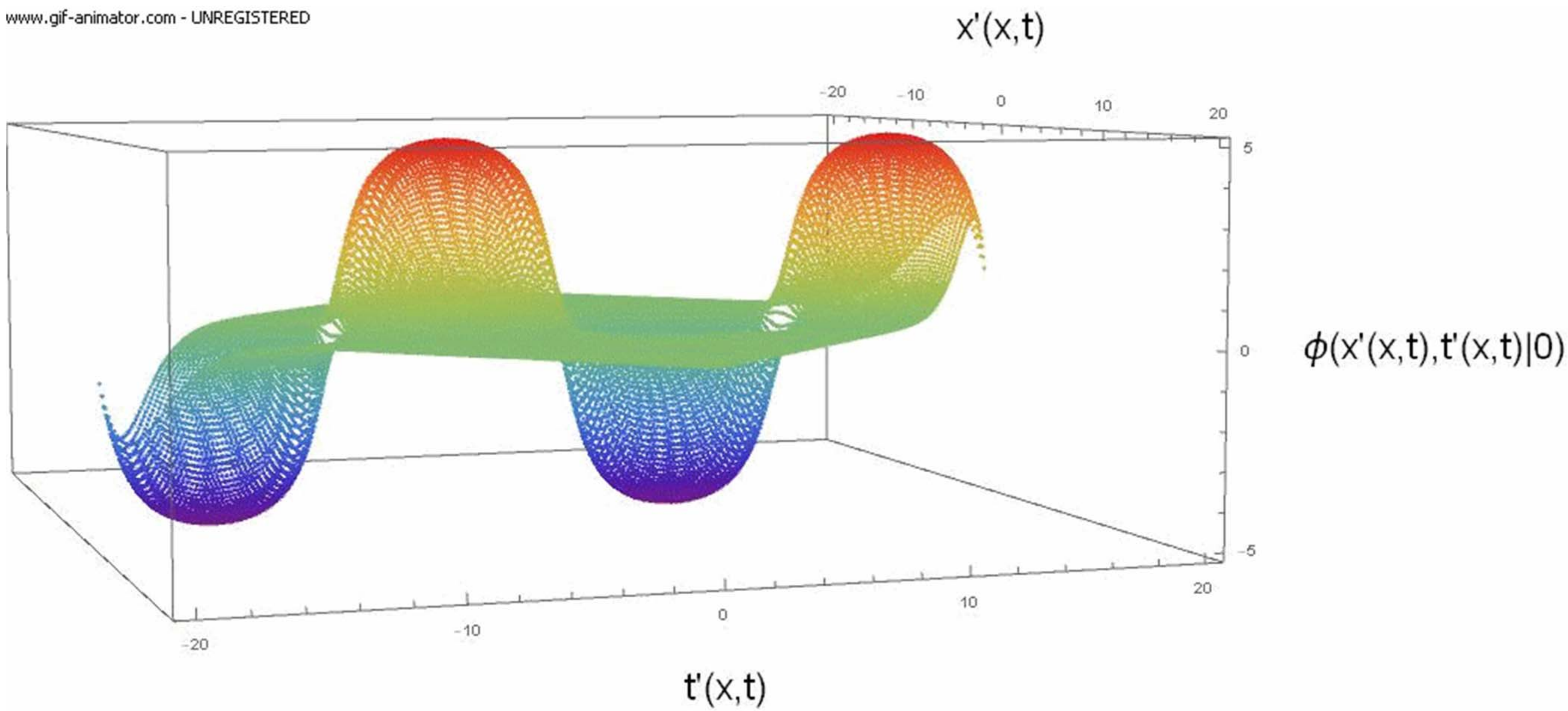
explained as

singularity of the change of variables

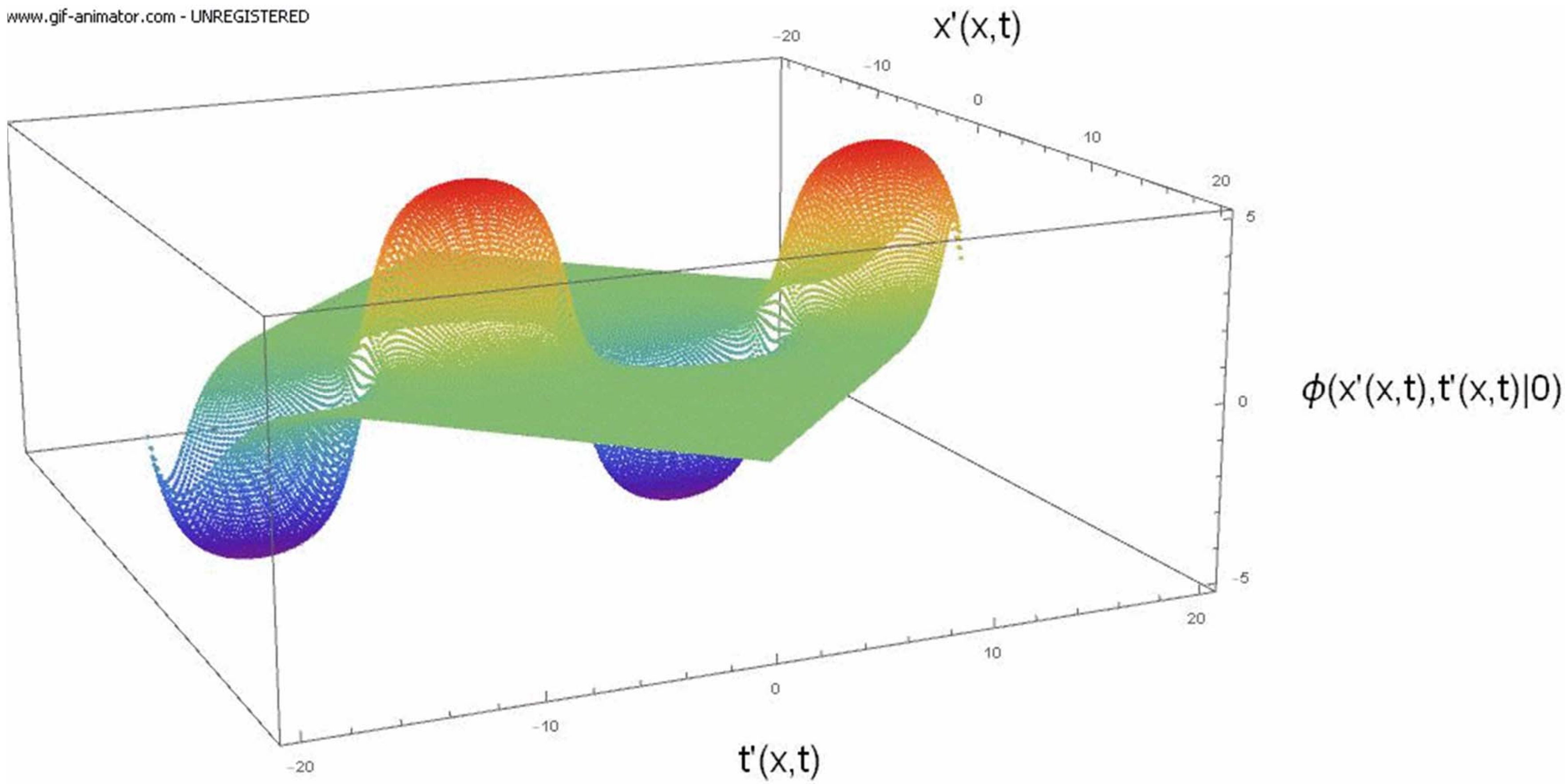
typical phenomenon of Burgers equation



Deformed breather solution (dilation)



Deformed breather solution (contraction)



Conti, Iannella, Negro, Tateo, arXiv:1806.11515 [hep-th]

Future directions

- Extension to higher dimensions : 4D Maxwell Born-Infeld theory (*non-linear electrodynamics*) as a $\mathbf{T\overline{T}}$-deformation of Maxwell electrodynamics

$$\frac{\partial}{\partial\tau}\mathcal{L}_{\mathrm{MBI}} = \sqrt{\det[T_{\mathrm{MBI}}]} = \frac{1}{4}\left(\frac{1}{2}\mathrm{Tr}[T_{\mathrm{MBI}}]^2 - \mathrm{Tr}\left[T_{\mathrm{MBI}}^2\right]\right)$$

$$\mathcal{L}_{\mathrm{MBI}}(\tau) = \frac{1}{2\tau}\left(-\sqrt{\det\left[g_{\mu\nu}\right]} + \sqrt{\det\left[g_{\mu\nu} + \sqrt{2\tau}\,F_{\mu\nu}\right]}\right) \quad , \quad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

Goal : find a renormalization scheme to quantize Maxwell Born-Infeld.

Problem : no factorization property of $\mathbf{T\overline{T}}$ in dimension higher than 2!

List of publications, talks and posters

Publications :

1) D. Bombardelli, A. Cavaglià, R. Conti and R. Tateo, “Exploring the spectrum of planar AdS_4/CFT_3 at finite coupling”, JHEP 04 (2018) 117 arXiv:1803.04748 [hep-th].
2) R. Conti, L. Iannella, S. Negro and R. Tateo, “Generalised Born-Infeld models, Lax operators and the $\mathbf{T\overline{T}}$ perturbation”, arXiv:1806.11515 [hep-th].
3) R. Conti, S. Negro, R. Tateo, “The $\mathbf{T\overline{T}}$ perturbation and its geometric interpretation”, arXiv:1809.09593 [hep-th].
4) D. Bombardelli, A. Cavaglià, R. Conti, R. Tateo, “On the analytic structure of anomalous dimensions in planar AdS_4/CFT_3”, (to appear).

Talks and posters :

1) Talk at “Young Researcher Integrability School and Workshop 2017” with the title “Numerical results in ABJM theory”.
2) Poster at “Young Researcher Integrability School and Workshop 2018” with the title “Exploring the spectrum of AdS_4/CFT_3 at finite coupling”.
3) Poster at “Integrability in Gauge and String Theory 2018” with the title “Generalised Born-Infeld models, Lax operators and the $\mathbf{T\overline{T}}$ perturbation”.