DYNAMICS OF DISK MASS SURVEY GALAXIES IN *REFRACTED GRAVITY*

VALENTINA CESARE

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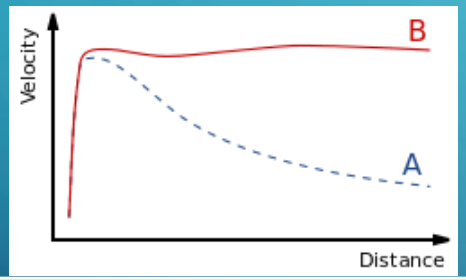
1. MOTIVATION OF THE ANALYSIS

Coma cluster NASA's Spitzer Space Telescope.

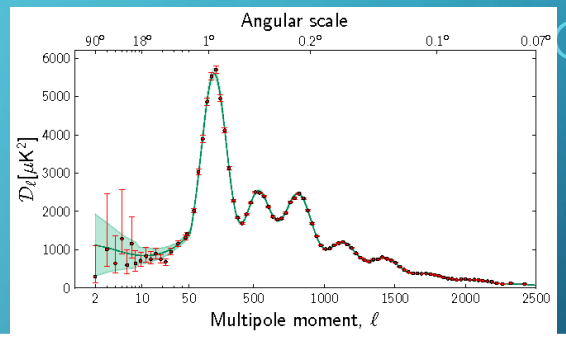


$v(R)$ trend expected (A, keplerian fall) and observed (B) wikipedia.org.

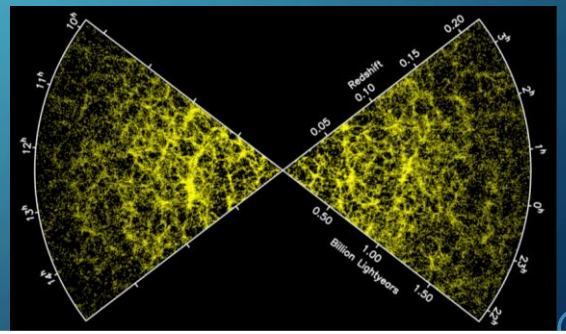
MASS DISCREPANCY PROBLEM

DARK MATTER

MODIFIED GRAVITY



CMB power spectrum physics.stackexchange.com.

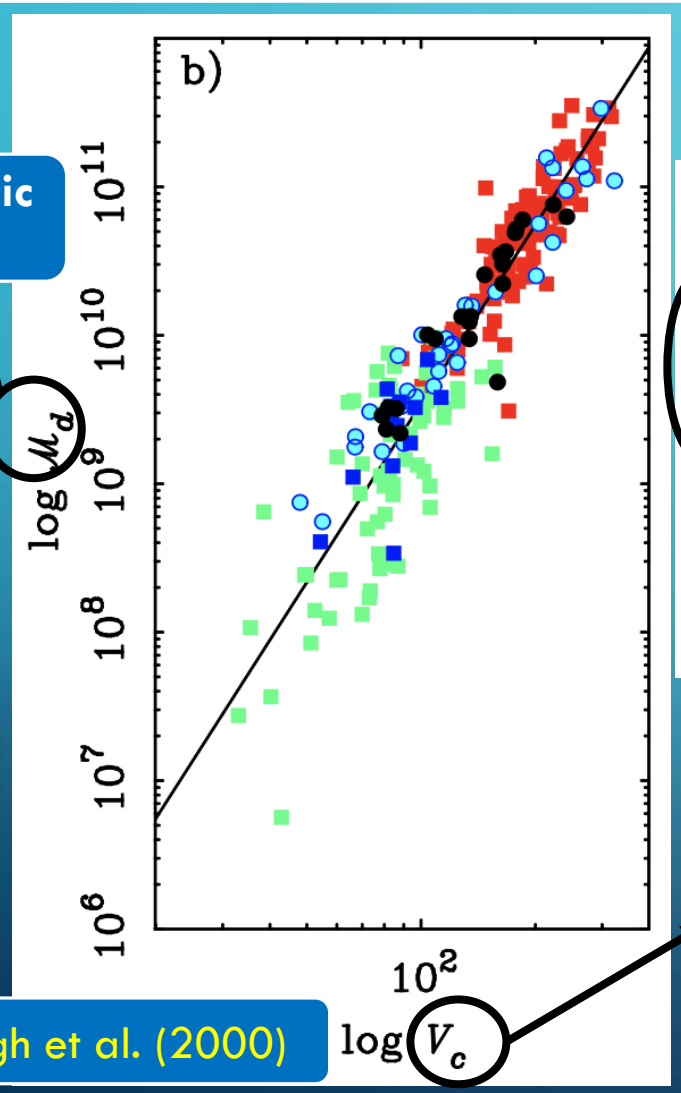


Large scale structure roe.ac.uk.

WHY CONSIDERING MODIFIED GRAVITY?

BARYONIC TULLY FISHER RELATION

$$\left(\frac{v_{\text{observed}}}{v_{\text{baryonic}}}\right)^2$$

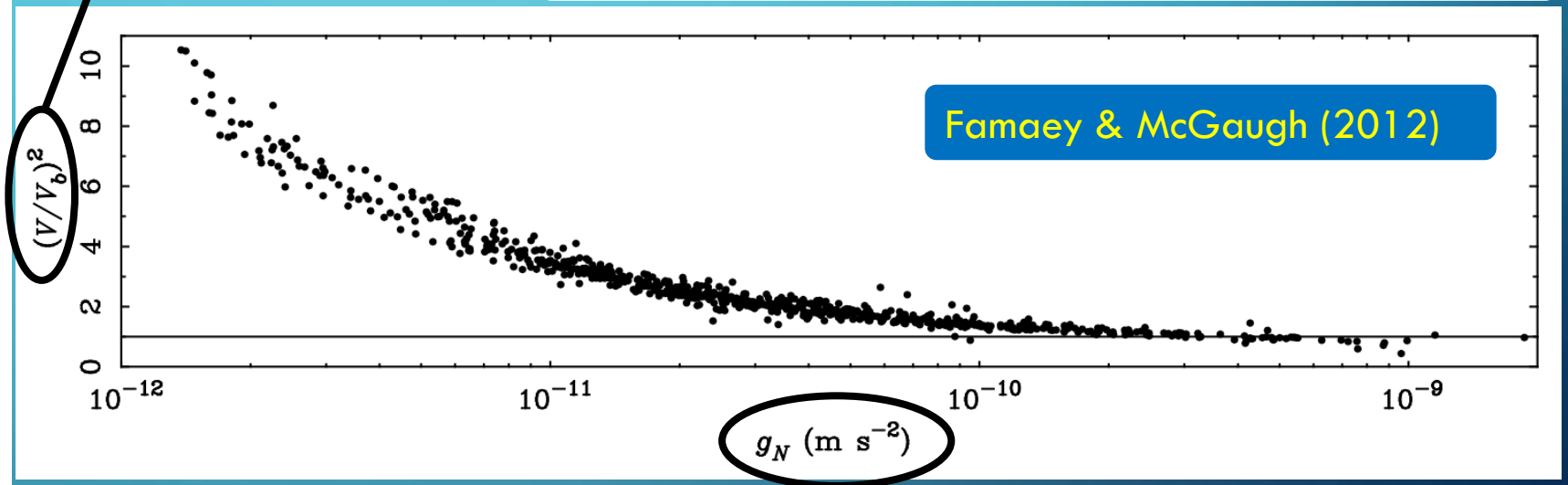


Baryonic mass

McGaugh et al. (2000)

Asymptotic flat rotation velocity

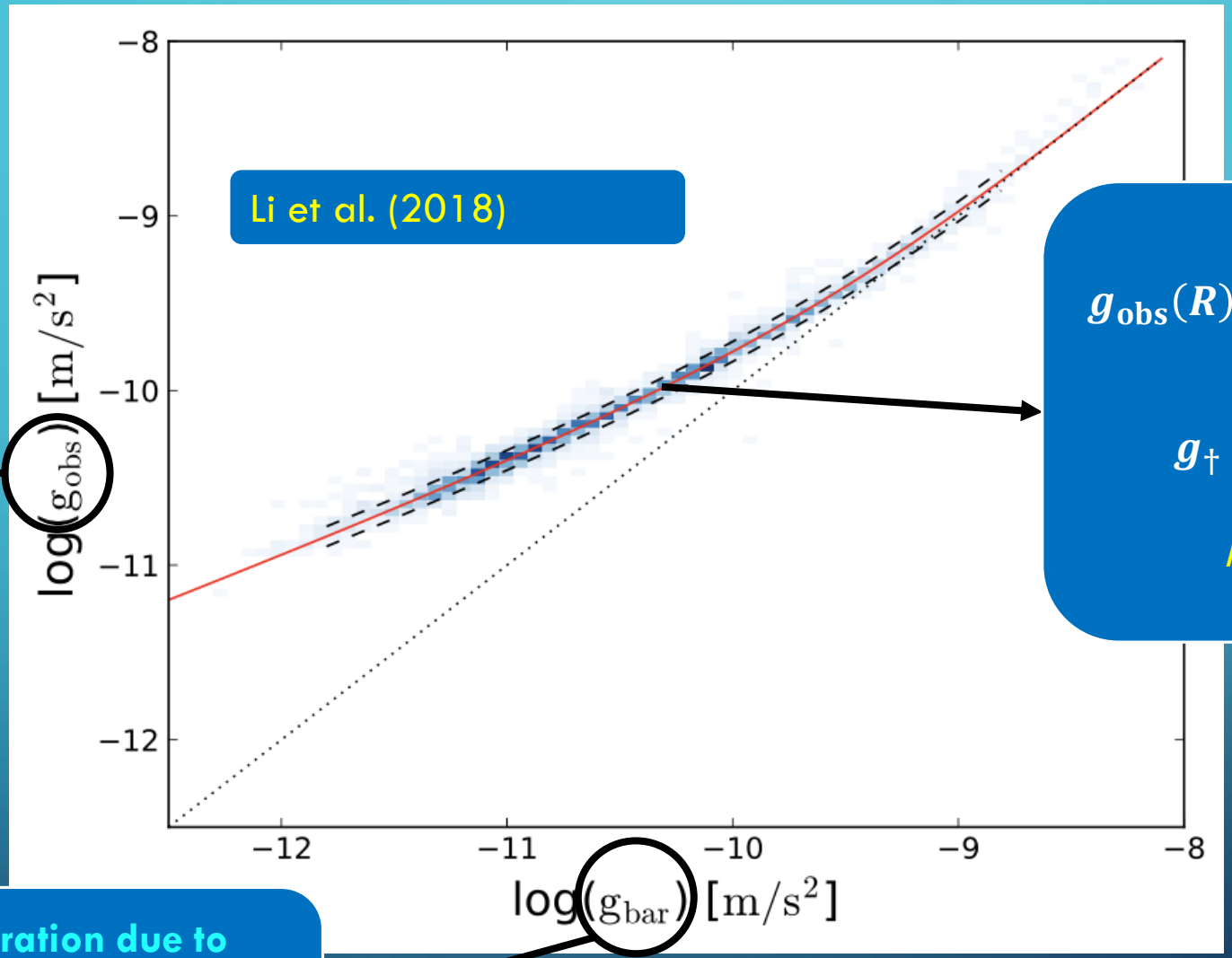
MASS DISCREPANCY ACCELERATION RELATION



Newtonian acceleration due to baryons

$$\frac{\partial \phi_{\mathrm{N}}}{\partial R} \leftarrow \nabla^2 \phi_{\mathrm{N}} = 4\pi G \rho$$

RADIAL ACCELERATION RELATION



$$\frac{v_{\mathrm{obs}}(R)^2}{R}$$

$$g_{\mathrm{obs}}(R) = \frac{g_{\mathrm{bar}}(R)}{1 - \exp\left(-\sqrt{g_{\mathrm{bar}}(R)/g_\dagger}\right)}$$

$$g_\dagger = 1.20 \times 10^{-10}\ \mathrm{m/s^2}$$

McGaugh et al. (2016)

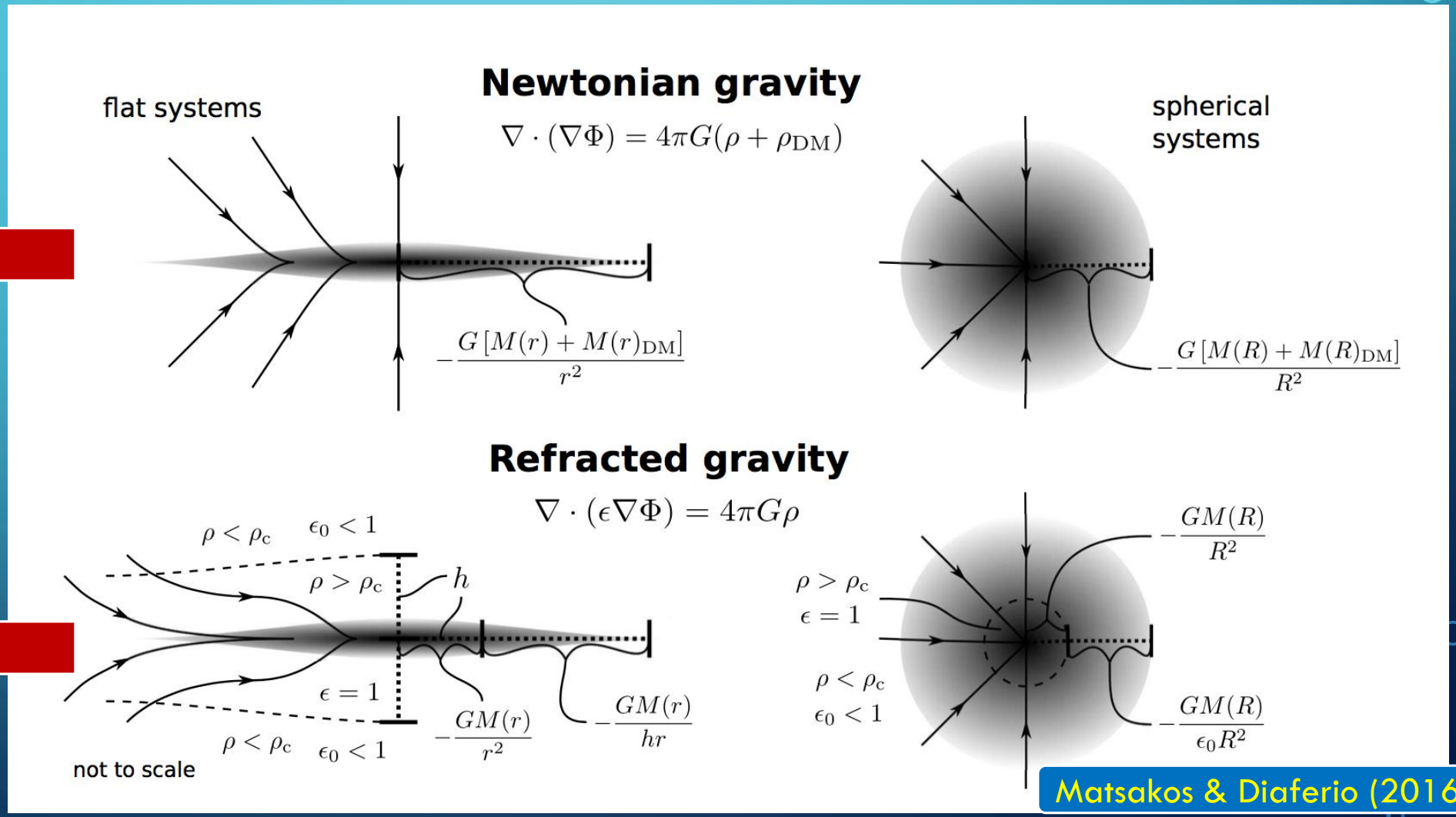
Newtonian acceleration due to baryons

$$\frac{\partial \phi_{\mathrm{N}}}{\partial R} \leftarrow \nabla^2 \phi_{\mathrm{N}} = 4\pi G \rho$$

2. REFRACTED GRAVITY

DM PRESENCE

FIELD LINES FOCUSSING



Matsakos & Diaferio (2016)

MODIFIED POISSON EQUATION

$$\nabla \cdot [\epsilon(\rho)\nabla\phi] = 4\pi G\rho$$

GRAVITATIONAL PERMITTIVITY

Matsakos & Diaferio (2016)

1 for $\rho \gg \rho_c$

ϵ_0 for $\rho \ll \rho_c$

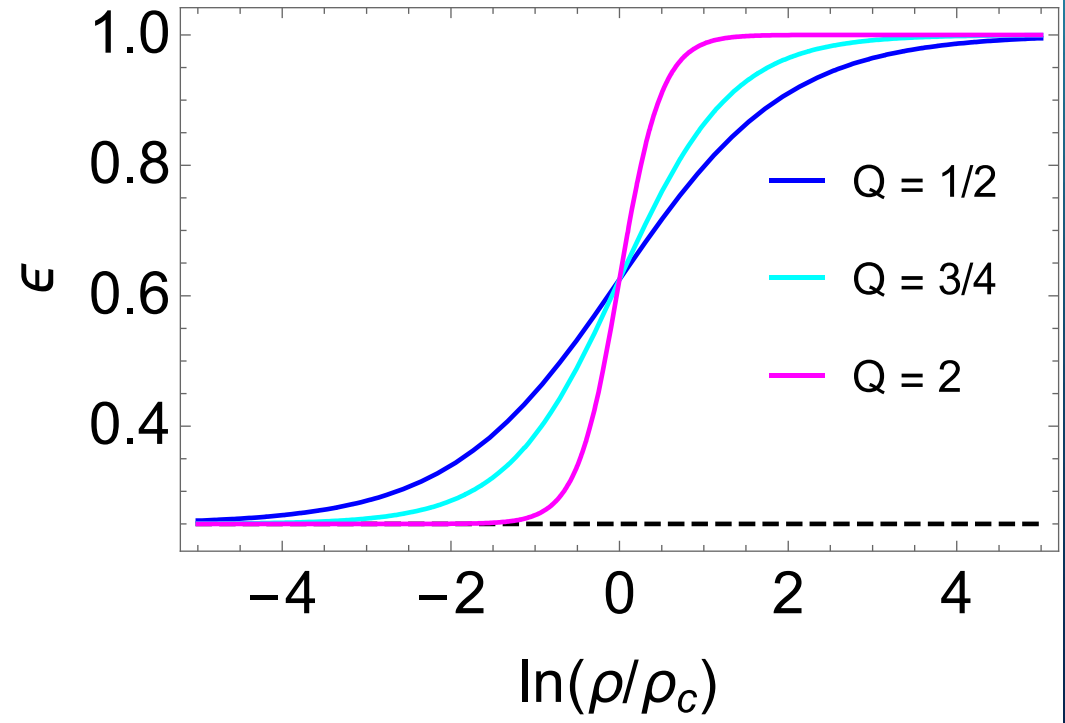
$0 \leq \epsilon_0 \leq 1$

Arbitrary choice for the gravitational permittivity $\epsilon(\rho)$

$$\epsilon(\rho) = \epsilon_0 + (1 - \epsilon_0)\frac{1}{2}\left\{\tanh\left[\ln\left(\frac{\rho}{\rho_c}\right)^{Q}\right] + 1\right\}$$

with $\{\epsilon_0, Q, \rho_c\}$ free universal parameters

Matsakos & Diaferio (2016)



Cesare et al., submitted

3. ROTATION CURVES AND VERTICAL VELOCITY DISPERSIONS

- Analysis in **Cesare et al., submitted**
- 30 disk galaxies from the **Disk Mass Survey (DMS)** (Bershady et al. 2010a)
- **Density model**:

  a) **Stellar disk**: $\rho_{\mathrm{d}}(R,z) = \frac{\Upsilon}{2h_z} I_{\mathrm{d,interp}}(R) \exp\left(-\frac{|z|}{h_z}\right)$

  b) **Spherical stellar bulge**: $\rho_{\mathrm{b}}(r) = -\frac{\Upsilon}{\pi}\int_r^{+\infty} \frac{dI_{\mathrm{b}}(R)}{dR}\frac{dR}{\sqrt{R^2-r^2}}$ where

  $$I_{\mathrm{b}}(R) = I_{\mathrm{e}} \exp\left\{-7.67\left[\left(\frac{R}{R_{\mathrm{e}}}\right)^{1/n_{\mathrm{s}}} - 1\right]\right\}$$

  c) **Atomic and molecular gas**: $\rho_{\mathrm{atom,mol}}(R,z) = \Sigma_{\mathrm{atom,mol,interp}}(R)\,\delta(z)$

- Successive Over Relaxation (SOR) **Poisson solver** to obtain RG potential
- **MCMC** to estimate the M/L, Υ, the disk scale height, h_z, and the three RG parameters, ϵ_0, Q and ρ_c
  - ➢ From rotation curves
  - ➢ From rotation curves and vertical velocity dispersions

UGC 7917

Bershady et al. 2010a

$$\nabla \cdot [\epsilon(\rho)\nabla\phi] = 4\pi G\rho$$

$$\phi$$

ROTATION CURVE

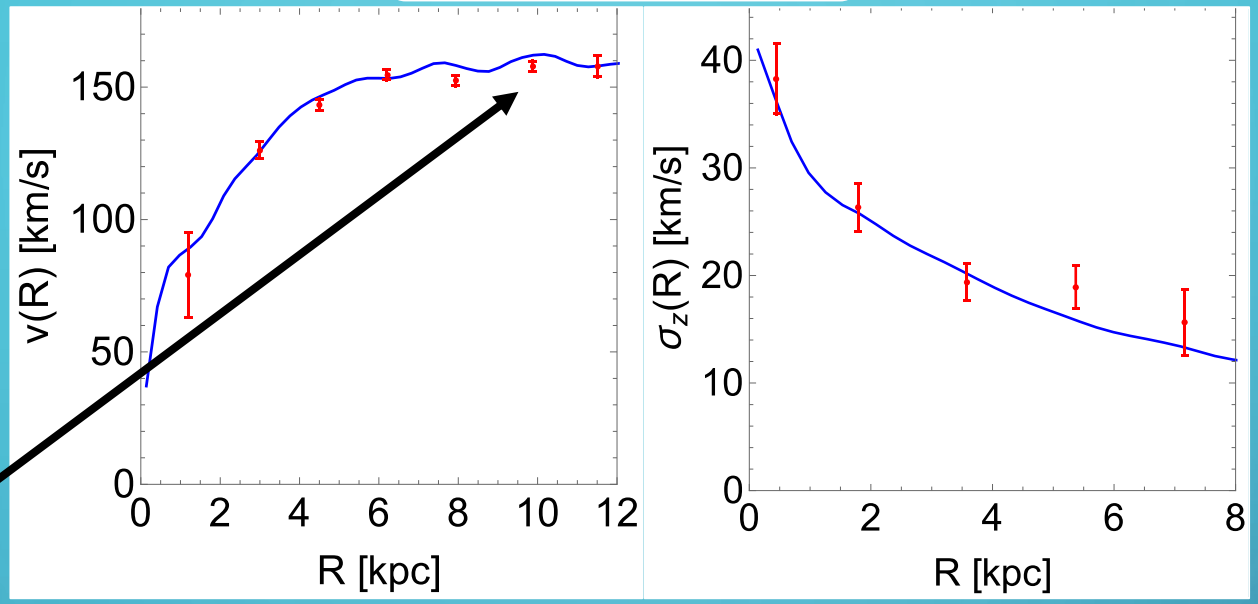
$$v(R, z=0) = \sqrt{R\frac{\partial\phi(R, z=0)}{\partial R}}$$

VERTICAL VELOCITY DISPERSION PROFILE

$$\sigma_z^2(R) = \frac{1}{h_z}\int_0^{+\infty}\left[\int_z^{+\infty}\exp\left(-\frac{|z'|}{h_z}\right)\frac{\partial\phi(R, z')}{\partial z'}dz'\right]dz$$

From Jeans analysis (Nagai & Miyamoto 1976; Nipoti et al. 2007)

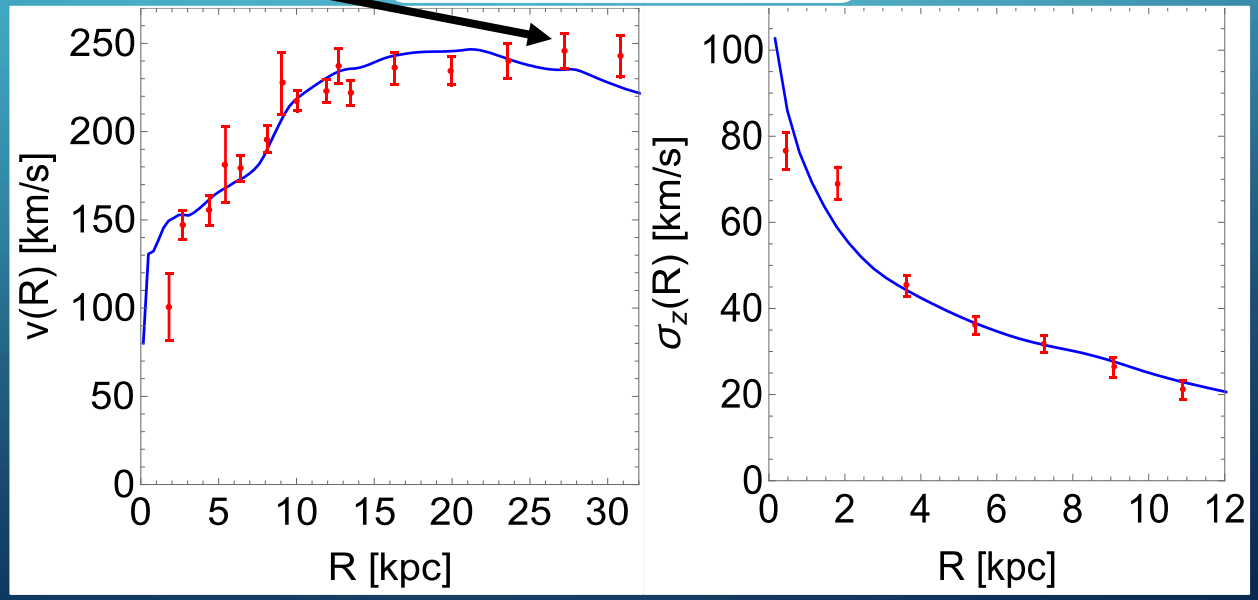
UGC 3091



Flat trend recovered

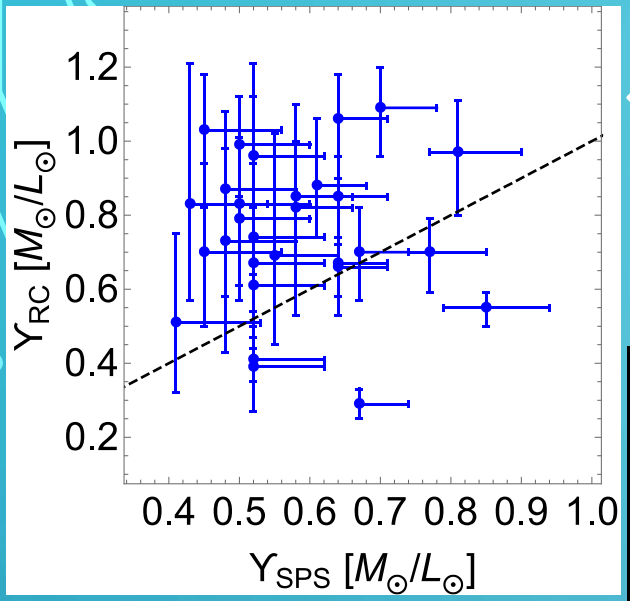
Renzo's rule

UGC 4256



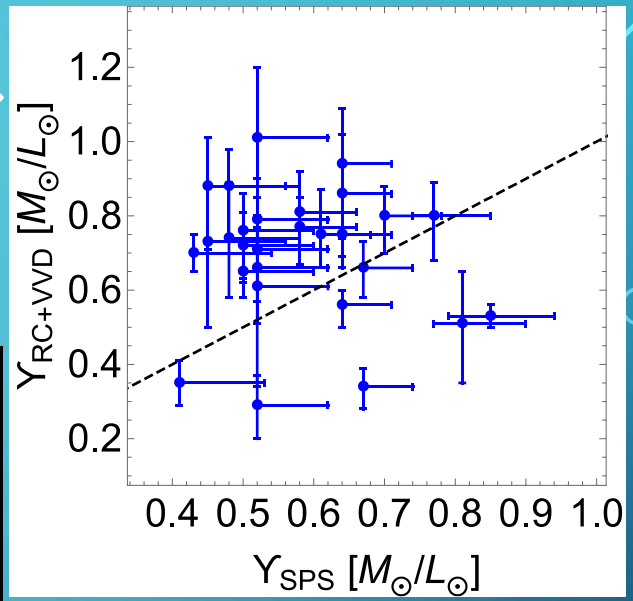
Good description of kinematic profiles

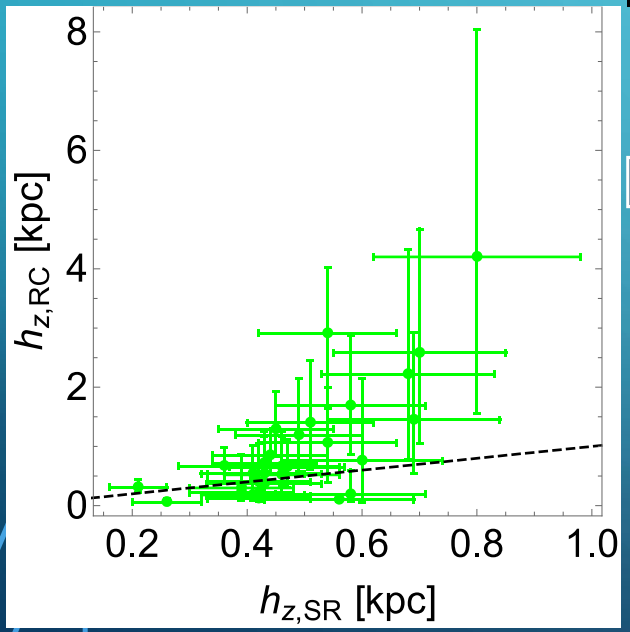
Cesare et al., submitted



Good agreement with SPS models in both cases

Xilouris et al. (1999)



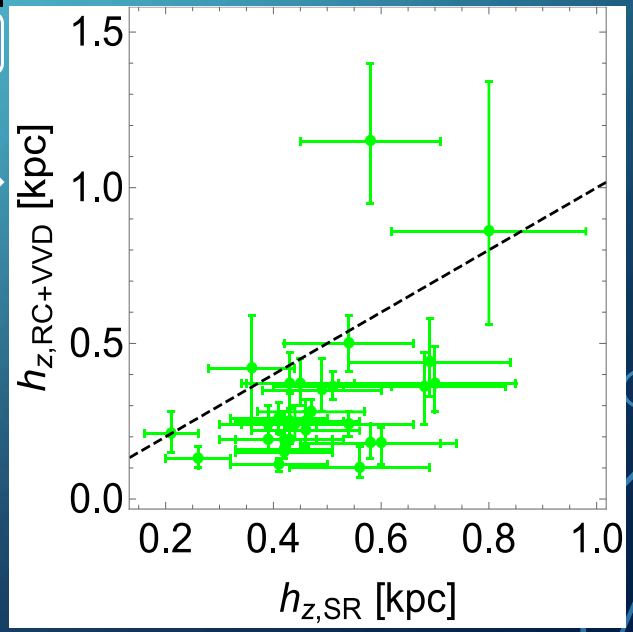


h_z decreases

Same result in Angus et al. (2015) with QUMOND

Observational bias

(Milgrom 2015, Aniyan et al. 2016)



Cesare et al., submitted

4. A UNIVERSAL COMBINATION OF RG PARAMETERS

Mean RG parameter from previous analysis

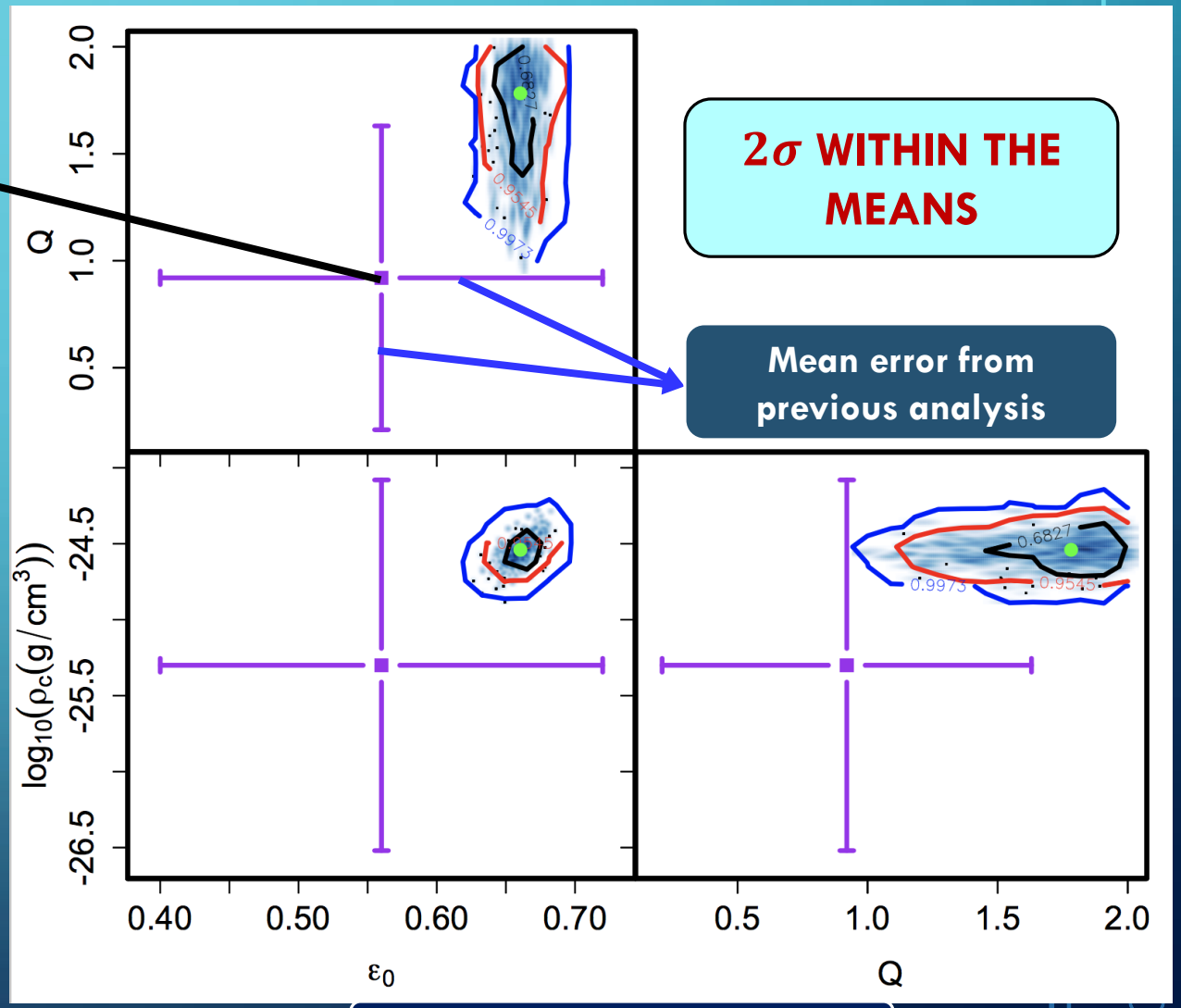
- $\{\epsilon_0, Q, \rho_c\}$ **FREE UNIVERSAL PARAMETERS**

IDEAL APPROACH

- Exploration of the 63-dimensional parameter space with the MCMC: $\{\epsilon_0, Q, \rho_c\} + 2 \times 30\,\{\Upsilon, h_z\}$

ADOPTED APPROACH

- Υ and h_z estimated with the previous analysis
- Exploration of the 3-dimensional parameter space $\{\epsilon_0, Q, \rho_c\}$ with the MCMC



2σ WITHIN THE MEANS

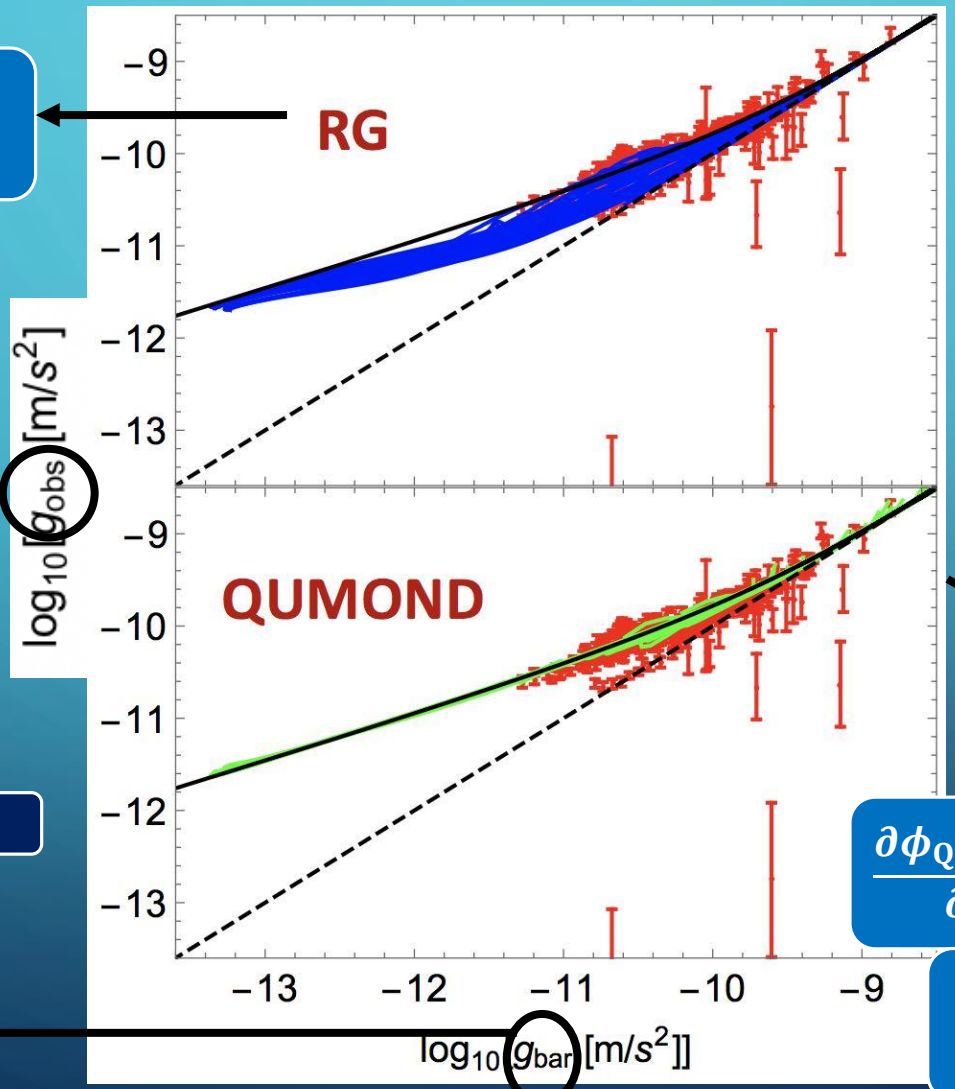
Mean error from previous analysis

Cesare et al., submitted

5. THE RADIAL ACCELERATION RELATION

$$\frac{\partial \phi_{\mathrm{RG}}}{\partial R} \leftarrow \nabla \cdot [\epsilon(\rho) \nabla \phi_{\mathrm{RG}}] = 4\pi G \rho$$

$$\frac{v_{\mathrm{obs}}(R)^2}{R}$$



SCATTER OF THE RAR

$$\frac{\partial \phi_{\mathrm{QUMOND}}}{\partial R} \leftarrow \nabla^2 \phi_{\mathrm{QUMOND}} = \nabla \cdot \left[\nu \left(\frac{|\nabla \phi_{\mathrm{N}}|}{a_0} \right) \nabla \phi_{\mathrm{N}} \right]$$

$$\nu \left(\frac{|\nabla \phi_{\mathrm{N}}|}{a_0} \right) = \frac{1}{2} \left(1 + \sqrt{1 + \frac{4}{|\nabla \phi_{\mathrm{N}}|/a_0}} \right)$$

Cesare et al., submitted

$$\frac{\partial \phi_{\mathrm{N}}}{\partial R} \leftarrow \nabla^2 \phi_{\mathrm{N}} = 4\pi G \rho$$

6. DYNAMICS OF ELLIPTICAL GALAXIES: A NEW PROJECT

- Description of the dynamics of 260 elliptical galaxies from the **ATLAS^{3D} survey** (Cappellari et al. 2011) with the **axisymmetric Jeans Anisotropic MGE (JAM) model** (Cappellari 2008)
- Density model: **Multi Gaussian Expansion (MGE)** (Emsellem et al. 1994)
- Comparison of the **mass-to-light ratios** with SPS model predictions and of the **RG parameters** with those extimated from DMS galaxies
- Test of the correlation between elliptical galaxies ellipticities and mass-to-light ratios (Deur 2014) predicted by RG

7. CURRENT/FUTURE PERSPECTIVES OF THE PROJECT

- Covariant formulation of the theory (Sanna et al. in preparation)
- Linear perturbation theory for the density field
- Power spectrum of the CMB anisotropies
- Formation and evolution of cosmic structures (N-body simulations, galaxy clusters)
- Elliptical and dwarfs galaxies

8. CONCLUSIONS

- RG properly reproduces the kinematics of DMS galaxies
- Introducing the vertical velocity dispersions we obtain disk scale heights smaller than observations → **observational bias, not issue of the theory**
- A unique combination of $\{\epsilon_0, Q, \rho_c\}$ is likely to be found to properly describe DMS kinematic profiles
- RG predicts a RAR with too much intrinsic scatter but it properly describes the fitting RAR asymptotic limits and the measured data → further investigation with **SPARC** (Lelli et al. 2016)
- RG can compete with other theories of gravity to describe the dynamics on galactic scale, deserving further investigation → **elliptical galaxies**

THANK YOU FOR THE ATTENTION! ☺

ADDITIONAL SLIDES

A. MORE INFORMATION ABOUT RG

SPHERICAL SYSTEMS OF MASS $M(<r)$

$$\nabla \cdot [\epsilon(\rho)\nabla\phi] = 4\pi G\rho \rightarrow \frac{\partial\phi}{\partial r} = \frac{1}{\epsilon(\rho)}\frac{GM(<r)}{r^2}$$

Enhanced by $\frac{1}{\epsilon(\rho)}$ compared to Newton

Same direction and dependence as in Newton → $g \propto r^{-2}$ in external regions

NOT SPHERICALLY SYMMETRIC SYSTEMS

$$\nabla \cdot [\epsilon(\rho)\nabla\phi] = 4\pi G\rho \rightarrow \frac{\partial\epsilon}{\partial\rho}\nabla\rho\cdot\nabla\phi + \epsilon\nabla^2\phi = 4\pi G\rho$$

ϕ depends on the variation of ρ

ϕ depends on ρ

Redirection of field lines → $g \sim (g_N a_0)^{1/2} \propto R^{-1}$ in external regions of flat systems ← MOND limit

B. THE POISSON SOLVER

Poisson equation is an elliptical Partial Differential Equation (PDE)

$$\nabla^2\phi(R,z) + S(\rho;R,z) = 0$$

Cylindrical coordinates

SOURCE TERM

RG

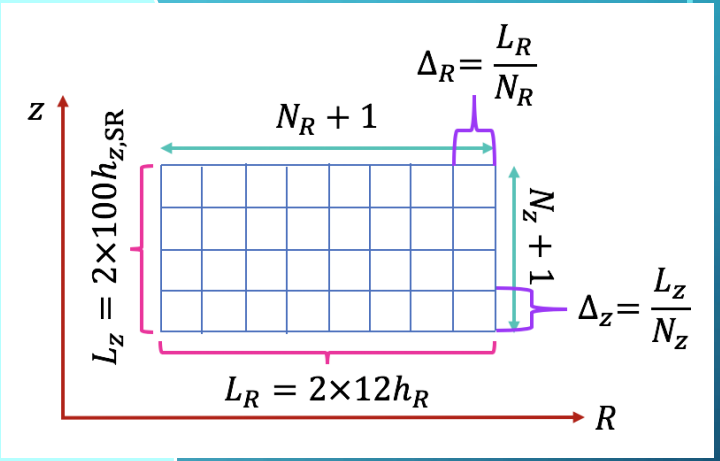
$$S_{\mathrm{RG}}(R,z) = -\frac{4\pi G\rho(R,z) - \nabla\epsilon(\rho)\cdot\nabla\phi_{\mathrm{RG}}(R,z)}{\epsilon(\rho)}$$

QUMOND

$$S_{\mathrm{QUMOND}}(R,z) = -\nabla\cdot\left[\nu\left(\frac{|\nabla\phi_{\mathrm{N}}|}{a_0}\right)\nabla\phi_{\mathrm{N}}\right]$$

$$S_{\mathrm{N}}(R,z) = -4\pi G\rho$$

- **Successive Over Relaxation (SOR)** Poisson solver, based on Jacobi and Gauss-Seidel algorithms (Young 1954).
- **PDE solving on a rectangular grid:**
  - **Dimension**: $(L_R = 2 \times 12h_R) \times (L_z = 2 \times 100h_{z,\mathrm{SR}})$
  - **Number of grid points**: $(N_R + 1) \times (N_z + 1)$
  - **Grid steps**: $\Delta_R = L_R/N_R,\ \Delta_z = L_z/N_z$



- Given $\phi_{i,k}^{n}$, $\phi_{i,k}^{n+1}$ is given by

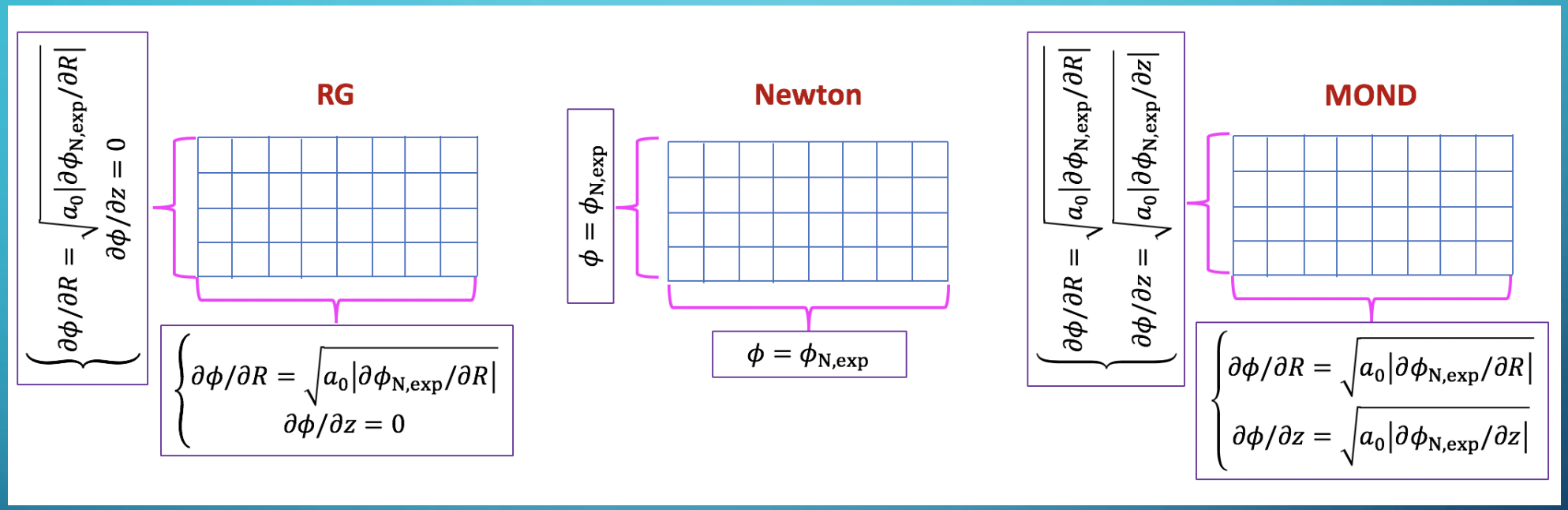
$$\phi_{i,k}^{n+1} = \phi_{i,k}^{n}(1-\omega_{\mathrm{SOR}}) + \frac{\omega_{\mathrm{SOR}}}{2R_i(\Delta_R^2+\Delta_z^2)}\left[\phi_{i+1,k}^{n+1}\left(R_i+\frac{1}{2}\Delta_R\right)\Delta_z^2 + \phi_{i-1,k}^{n+1}\left(R_i-\frac{1}{2}\Delta_R\right)\Delta_z^2 + (\phi_{i,k+1}^{n+1}+\phi_{i,k-1}^{n+1})R_i\Delta_R^2 + R_iS_{i,k}\Delta_R^2\Delta_z^2\right]$$

- ω_{SOR}: **convergence parameter**
  - $0 < \omega_{\mathrm{SOR}} < 2$: **convergence**
  - $1 < \omega_{\mathrm{SOR}} < 2$: **convergence faster than Gauss-Seidel**
  - $\omega_{\mathrm{SOR}} = 2/(1+\pi/\min\{N_R, N_z\}$: **fastest convergence**

- Iterations stop when

$$\varepsilon^{n+1} = \frac{1}{(N_R-1)(N_z-1)}\sum_{i,k}\left|\phi_{i,k}^{n+1}-\phi_{i,k}^{n}\right| < 10^{-9}$$

B.1 THE POTENTIAL BOUNDARY CONDITIONS



$$\phi_{\mathrm{N,exp}}(R,z) = -\frac{GM_1}{\sqrt{R^2 + \left(a_1 + \sqrt{z^2+b^2}\right)^2}} - \frac{GM_2}{\sqrt{R^2 + \left(a_2 + \sqrt{z^2+b^2}\right)^2}} - \frac{GM_3}{\sqrt{R^2 + \left(a_3 + \sqrt{z^2+b^2}\right)^2}}$$

Miyamoto & Nagai (1975), Smith et al. (2015)

C. THE MCMC ANALYSIS

PRIORS

Positively defined Gaussian priors for the mass parameters:

- $\boldsymbol{\Upsilon}$ $[\mathbf{M_\odot/L_\odot}]$: $\Upsilon_{\text{mean}} = \Upsilon_{\text{SPS}}$, $\Upsilon_{\text{st dev}} = 3 \times \max \text{err}_{\text{SPS}}$
- $\boldsymbol{h_z}$ $[\mathbf{kpc}]$: $h_{z,\text{mean}} = h_{z,\text{SR}}$, $h_{z,\text{st dev}} = 3 \times \text{err}_{h_{z,\text{SR}}}$
- $\boldsymbol{\Upsilon} > 0 \wedge \boldsymbol{h_z} > 0$

Flat priors for RG parameters in the following ranges:

- ϵ_0: $[0.10, 1.00]$
- Q: $[0.01, 2.00]$
- $\mathbf{log_{10}}[\rho_c\ (\mathbf{g/cm^3})]$: $[-27.00, -23.00]$

ACCEPTANCE CRITERION: METROPOLIS - HASTINGS

$$A = \left[\frac{p(x) \times \mathcal{L}(x)}{p(x_i) \times \mathcal{L}(x_i)}\right]\left[\frac{G(x|x_i)}{G(x_i|x)}\right]$$

≥ 1: x is accepted $\rightarrow$ $x_{i+1} = x$

< 1

- x is accepted $\rightarrow$ $x_{i+1} = x$ with probability A
- x is rejected $\rightarrow$ $x_{i+1} = x_i$ with probability $1 - A$

- $\vec{x} = (\Upsilon, h_z, \epsilon_o, Q, \log_{10}\rho_c)$ (or $\vec{x} = (\epsilon_o, Q, \log_{10}\rho_c)$ in the research of the universal combination of RG parameters)
- $G(x|x_i)$: **probability density** $\rightarrow$ multi-variate Gaussian density distribution with **mean = x_i** and **st dev = Gaussian prior st dev/3** (Υ, h_z) and **st dev = 10% of the prior range** ($\epsilon_o, Q, \log_{10}\rho_c$)
- $p(\vec{x})$: **prior**
- $\mathcal{L}(\vec{x})$: **likelihood**

$$\mathcal{L}(\vec{x}) = \exp\left[-\frac{\chi^2_{\text{red}}(\vec{x})}{2}\right]$$

$$\chi^2_{\text{red}}(\vec{x})$$

$$\chi^2_a(\vec{x}) = \frac{1}{N_{\text{RC}}-5}\sum_{i=1}^{N_{\text{RC}}}\frac{[v_{\text{mod}}(R_i;\vec{x})-v_{\text{data}}(R_i)]^2}{v^2_{\text{data,err}}(R_i)}$$

($v(R)$)

$$\chi^2_b(\vec{x}) = \left(\sum_{i=1}^{N_{\text{RC}}}\frac{[v_{\text{mod}}(R_i;\vec{x})-v_{\text{data}}(R_i)]^2}{v^2_{\text{data,err}}(R_i)} + \sum_{j=1}^{N_{\text{VVD}}}\frac{[\sigma_{z,\text{mod}}(R_j;\vec{x})-\sigma_{z,\text{data}}(R_j)]^2}{\sigma^2_{z,\text{data,err}}(R_j)}\right)/(N_{\text{RC}} + N_{\text{VVD}} - 5)$$

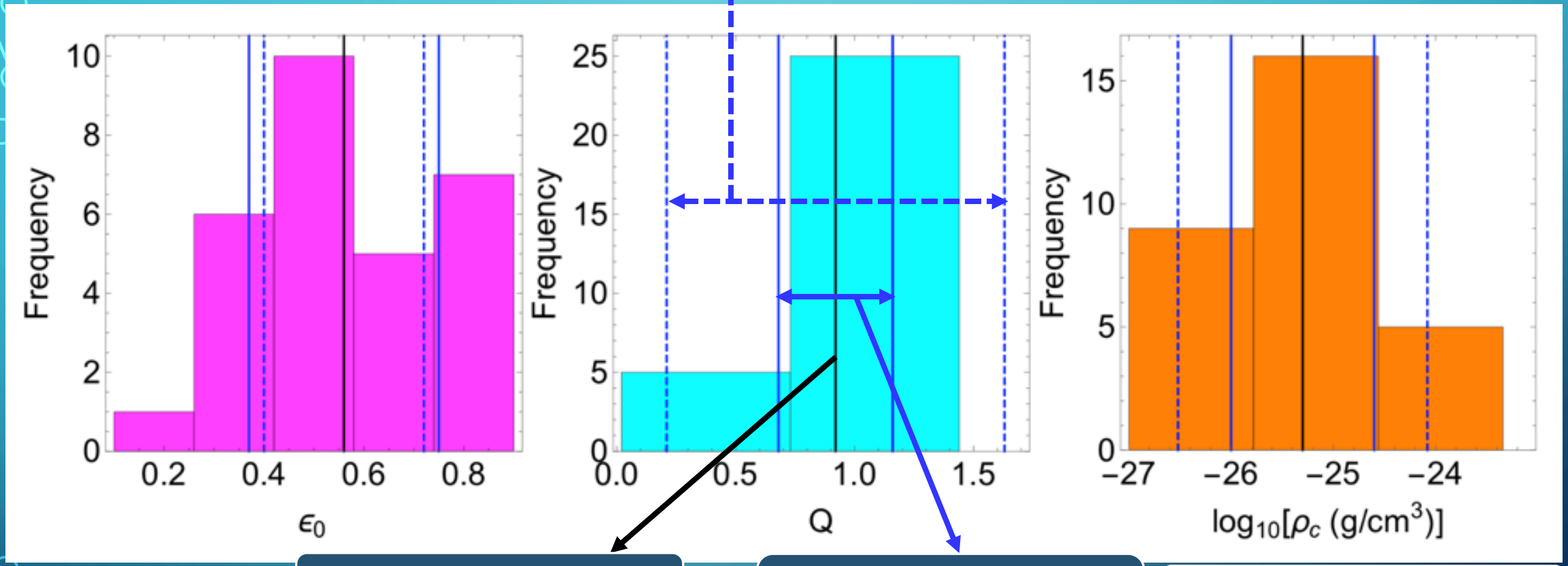
($v(R) + \sigma_z(R)$)

$$\chi^2_c(\vec{x}) = \sum_{k=1}^{N_{\text{gal}}}\left(\left(\sum_{i=1}^{N_{\text{RC},k}}\frac{[v_{\text{mod},k}(R_i;\vec{x})-v_{\text{data},k}(R_{,i})]^2}{v^2_{\text{data,err},k}(R_i)} + \sum_{j=1}^{N_{\text{VVD},k}}\frac{[\sigma_{z,\text{mod},k}(R_j;\vec{x})-\sigma_{z,\text{data},k}(R_j)]^2}{\sigma^2_{z,\text{data,err},k}(R_j)}\right)/(N_{\text{RC},k} + N_{\text{VVD},k} - 3)\right)$$

(**Universal combination of RG parameters**)

- **NUMBER OF STEPS OF THE CHAINS**: 19000 after the burn-in chain
- **NUMBER OF BURN - IN**: 1000
- **PARAMETERS ESTIMATORS**: medians of the posterior distributions
- **1σ UNCERTAINTY RANGE ON THE PARAMETERS ESTIMATORS**: range between the 15.9 and 84.1 percentiles $\rightarrow$ 68% of the posterior cumulative distribution centered on the median

Mean error from MCMC



Distribution mean

Distribution standard deviation

Cesare et al., submitted

Mean errors from MCMC $\gtrsim$ standard deviations of the RG parameters distributions

UNIVERSALITY OF RG PARAMETERS SUPPORTED

D. DERIVATION OF THE VERTICAL VELOCITY DISPERSION MODEL

- **Jeans equation** (Nagai & Miyamoto 1976; Nipoti et al. 2007):

$$\frac{\partial[\rho_*(R,z)\sigma^2(R,z)]}{\partial z} + \rho_*(R,z)\frac{\partial\phi(R,z)}{\partial z} = 0$$

with $\rho_*(R,z) \simeq \rho_{\mathrm{d}}(R,z) = \frac{\Upsilon}{2h_z} I_{\mathrm{d,interp}}(R)\exp\left(-\frac{|z|}{h_z}\right)$

- $L_{\mathrm{b}}/L_{\mathrm{d}} \sim 0.09 \Rightarrow$ **bulge contribution neglected**
- **Observed vertical velocity dispersion profile, weighted by $\Sigma_*(R)$**

$$\sigma_z^2(R) = \frac{1}{\Sigma_*(R)}\int_{-\infty}^{+\infty}\rho_*(R,z)\sigma^2(R,z)dz$$

- **Local stellar surface density**

$$\Sigma_*(R) = \int_{-\infty}^{+\infty}\rho_*(R,z)dz = \Upsilon I_{\mathrm{d,interp}}(R)$$

- **Resulting profile**

$$\sigma_z^2(R) = \frac{1}{h_z}\int_0^{+\infty}\left[\int_z^{+\infty}\exp\left(-\frac{|z'|}{h_z}\right)\frac{\partial\phi(R,z')}{\partial z'}dz'\right]dz$$

E. THE OBSERVATIONAL BIAS

VERTICAL VELOCITY DISPERSIONS σ_z

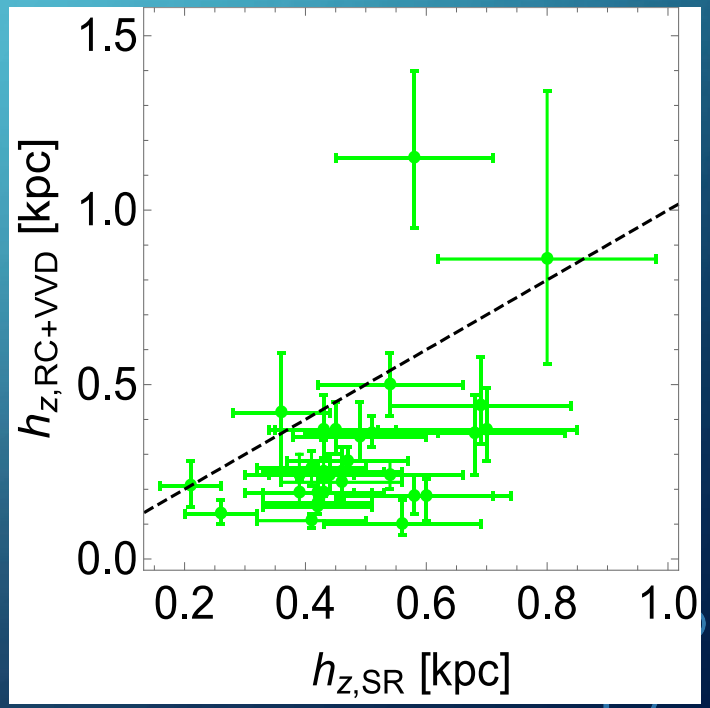
- Derived in ~ face-on galaxies
- Measured near the V-band, from integrated spectra
- Signal dominated by the younger stellar population ($\sigma_z \downarrow$, $h_z \downarrow$) → $h_{z,\mathrm{RC+VVD}}$ dominated by the younger stellar population

DISK SCALE HEIGHTS $h_{z,\mathrm{SR}}$

- Derived in edge-on galaxies
- Measured from near-IR photometry
- Signal derived from the older stellar population ($\sigma_z \uparrow$, $h_z \uparrow$)
- A scale relation between disk scale lengths and disk scale heights is inferred from edge-on galaxies (Bershady et al. 2010b)

$$\log_{10}\left(\frac{h_R}{h_{z,\mathrm{SR}}}\right) = 0.367\,\log_{10}\left(\frac{h_R}{\mathrm{kpc}}\right) + 0.708 \pm 0.095$$

Milgrom (2015), Aniyan et al. (2016)



Cesare et al., submitted

Aniyan et al. (2016) model a typical external face-on galaxy with the on-line Besançon model to quantify the ratio between the σ_z **of the hotter and older giant component alone** (σ_2) and the σ_z **of the entire stellar population** (σ_1)

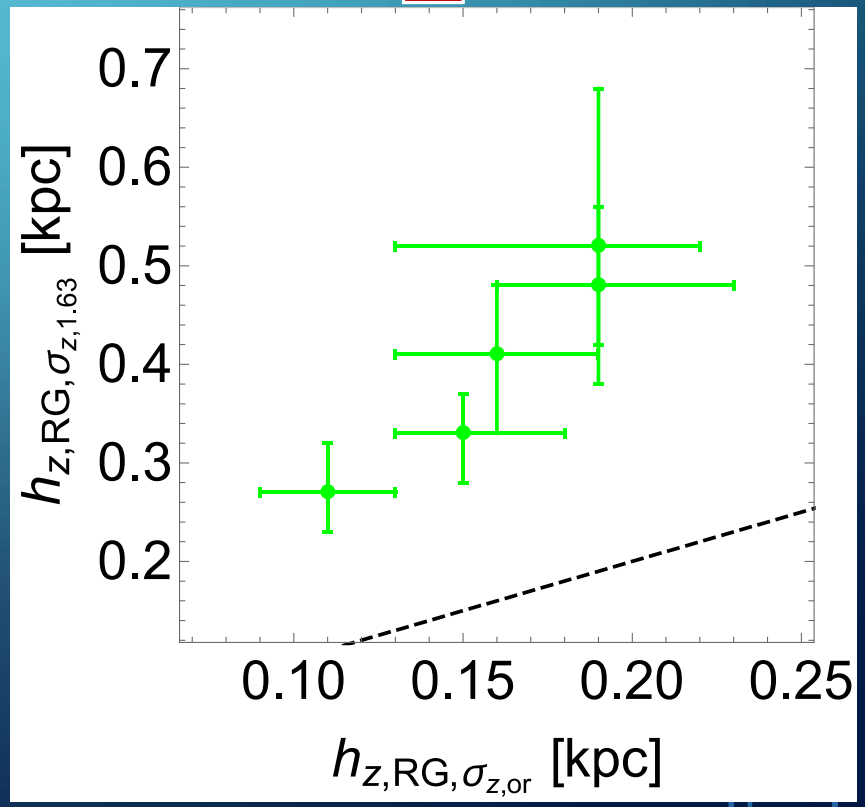
$$\eta = \frac{\sigma_2}{\sigma_1} = 1.55 \pm 0.02$$

Independent calculation of this factor:

- Vertical velocity dispersion computation with the parameters from rotation curves alone
- Intepolation at the radii of the measured vertical velocity dispersions profiles
- $\eta'_k = \left\langle \frac{\sigma_{z,\mathrm{interp}}}{\sigma_{z,\mathrm{meas}}} \right\rangle$ for every galaxy
- $\eta' = \langle \eta'_k \rangle \pm \mathrm{st\,dev}(\eta'_k) = 1.63 \pm 0.65$ between all galaxies
- Agreement between η' and η within $+0.12\sigma$.

h_z INCREASES BY ~ 2.5

Repetition of the MCMC analysis on $v(R) + \sigma_z(R)$ for five galaxies, increasing the measured $\sigma_z(R)$ by **1.63**.



Cesare et al., submitted

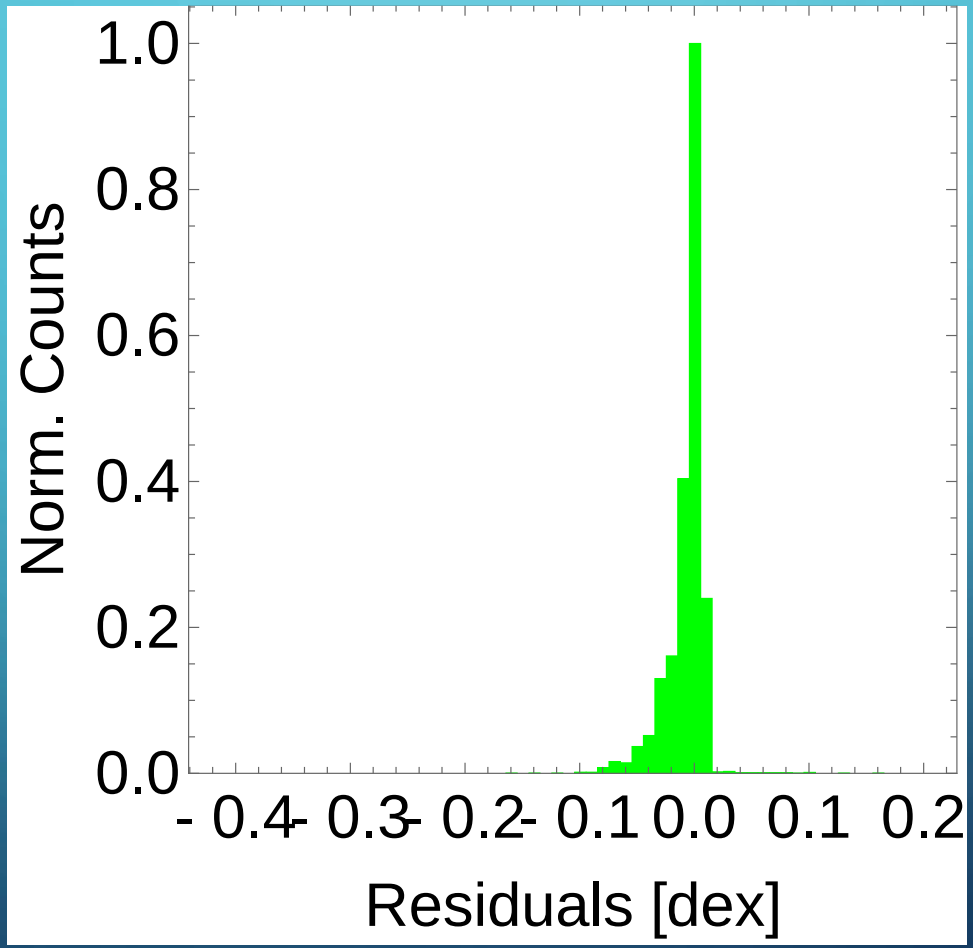
F. THE SCATTER OF THE RAR

Residuals computed from RAR fitting relation within

$$\log_{10}[g_{\mathrm{bar}}(\mathrm{m/s^2})] =$$

$$= [-11.28, -8.81],$$

range covered by the data



Cesare et al., submitted

OBSERVED SCATTER: 0.12 dex

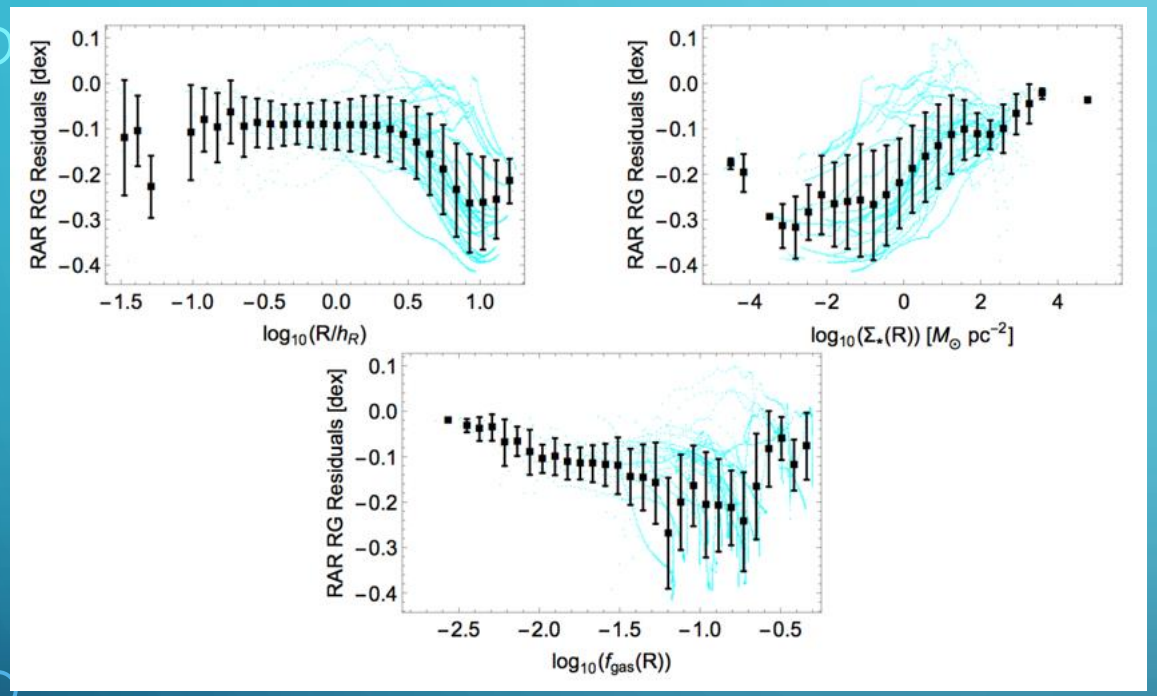
INTRINSIC SCATTER PREDICTED BY RG: 0.11 dex

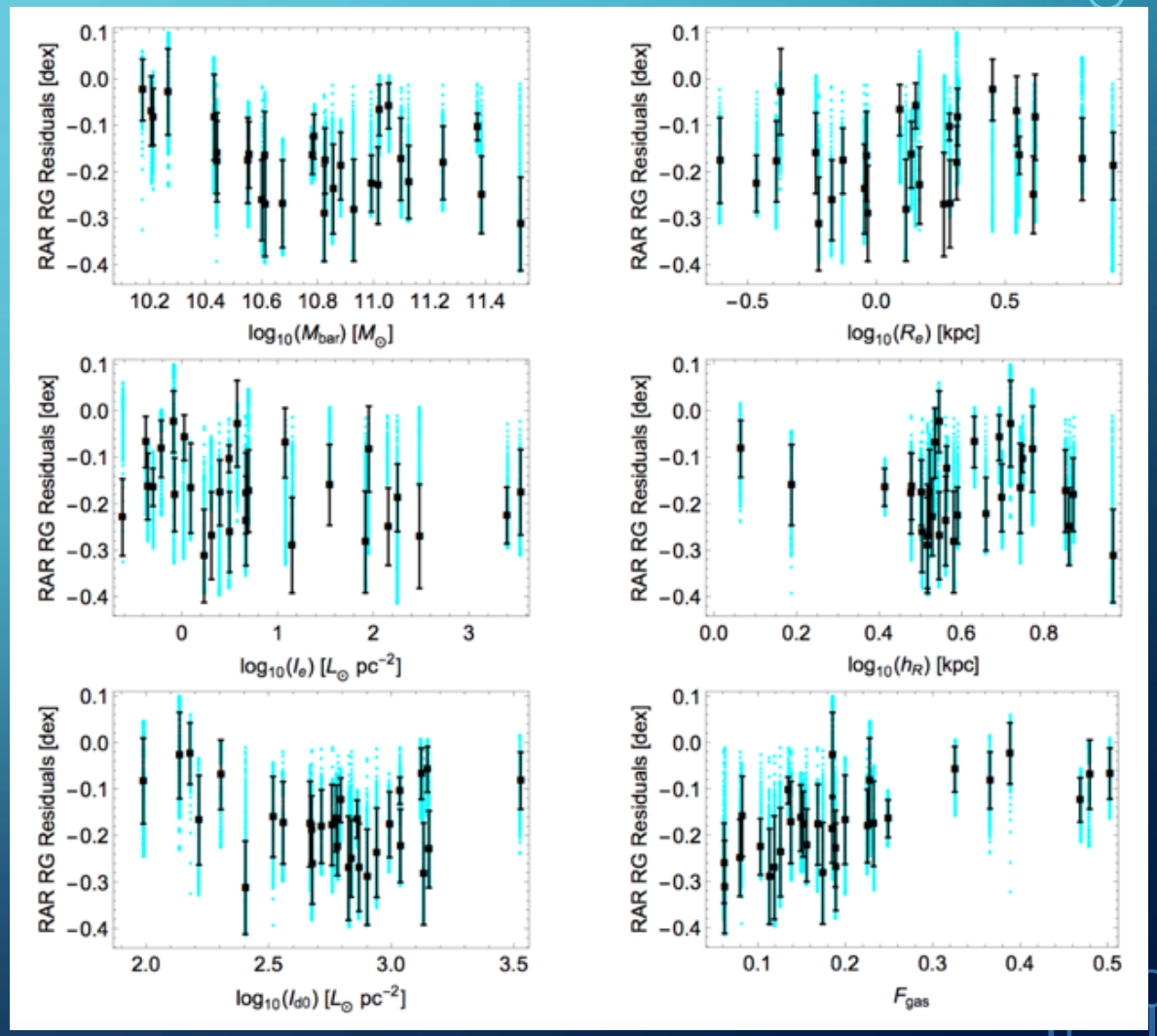
INTRINSIC SCATTER PREDICTED BY QUMOND: 0.017 dex

RG predicts too much intrinsic scatter

Analysis with a more suitable galaxy survey like **SPARC** (Lelli et al. 2016)

G. THE RESIDUALS OF THE RAR





RG residuals of the RAR seem to correlate with some galaxy properties

Analysis with a more suitable galaxy survey like **SPARC** (Lelli et al. 2016)