Exploration of the QCD phase diagram

Mario Motta

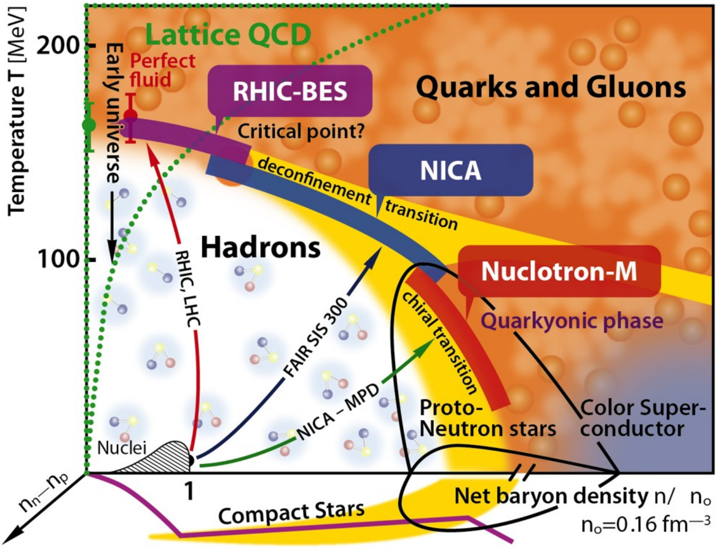
University of Turin

September 20 2019, Turin

Contents

Exploration of the QCD phase diagram

Phase Diagram



Why we explore the QCD Phase Diagram?

The description of nuclear matter and the interaction between nucleons in the nuclei should ultimately be provided by QCD. This theory contains two important features:

- confinement
- spontaneous chiral symmetry breaking

The knowledge of these two QCD features is not complete. the explorations of the phase diagram (in particular the region where chiral symmetry is restored and confinement does not occur) may provide a full understanding of these two phenomena

Observables

Which Observables are important for the explorations of the Phase Diagram?

- isentropic trajectories

Observables

Which Observables are important for the explorations of the Phase Diagram?

- isentropic trajectories
- speed of sound

Observables

Which Observables are important for the explorations of the Phase Diagram?

- isentropic trajectories
- speed of sound
- order parameters

Observables

Which Observables are important for the explorations of the Phase Diagram?

- isentropic trajectories
- speed of sound
- order parameters
- fluctuations of conserved charge of QCD

Isentropic trajectories

Along isentropic trajectories, the system evolves in time keeping constant entropy. The QGP expands, during this expansion baryon number density and entropy density change (pure dilution), but $s/n = cost$. Then the isentropic trajectories is replaced by "Iso-s/n" trajectories

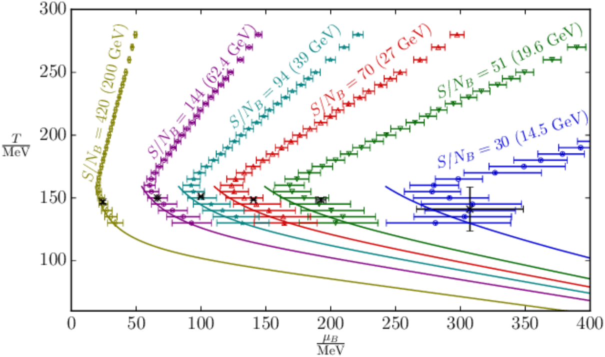
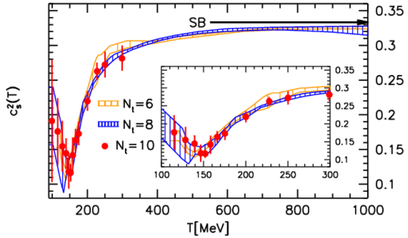


Figure: isentropic trajectories from N.Guenther et al Nucl.Phys. A967 (2017) 720-723

Speed of sound

The speed of sound is one of the most important characteristics in hydrodynamics: it is responsible for the collective acceleration of the fireball and it governs the evolution of the fireball produced in the heavy-ion collision as well as one of the most important observables for describing of QGP formation: the elliptic flow.



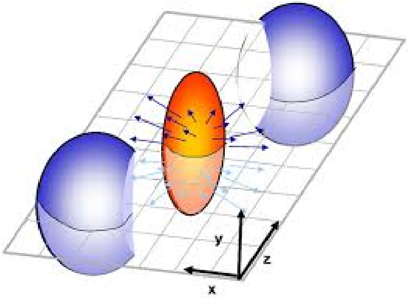


Figure: the square of speed of sound (Szabolcs Borsanyi et all JHEP 1011 (2010) 077) and a picture of HIC

Speed of sound

The speed of sound is one of the most important characteristics in hydrodynamics: it is responsible for the collective acceleration of the fireball and it governs the evolution of the fireball produced in the heavy-ion collision as well as one of the most important observables for describing of QGP formation: the elliptic flow.

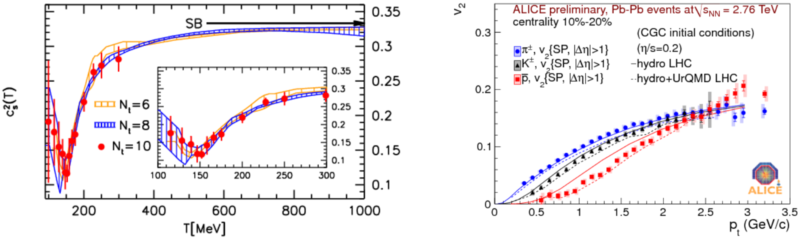


Figure: the square of speed of sound and the Elliptic Flow

The speed of sound is one of the most important characteristics in hydrodynamics: it is responsible for the collective acceleration of the fireball and it governs the evolution of the fireball produced in the heavy-ion collision as well as one of the most important observables for describing of QGP formation: the elliptic flow.

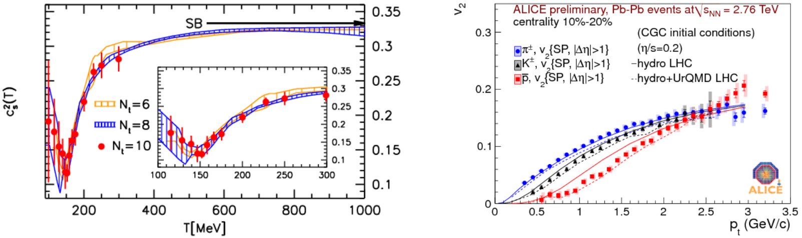


Figure: the square of speed of sound and the Elliptic Flow

$$(\quad + P)\frac{dv^i}{dt} = \quad c_s^2 \frac{d}{dx^i} \qquad (1)$$

Order Parameters

The order parameters signal the transition lines and their behaviour near the transition fixes the order of transition (cross-over, 1^{st}-order, 2^{st}-order,ect.)

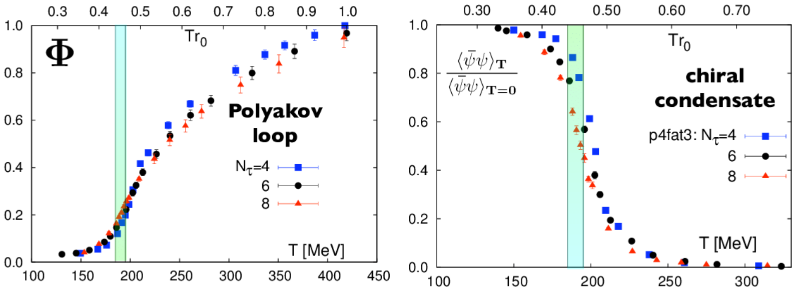


Figure: the order parameters for deconfinament and chiral symmetry restoration transition W.Weise,174,JPS,10.1143/PTPS.174.1

Fluctuations of QCD conserved charges

In many different fields, the study of fluctuations can provide physical insights into the underlying microscopic physics. The fluctuations can become invaluable physical observable in spite of their difficult character. Fluctuations are powerful tools to diagnose microscopic physics, to trace back the history of the system and the nature of its elementary degrees of freedom (see M. Asakawa and M. Kitazawa, Prog. Part. Nucl.Phys. 90, 299 (2016) doi:10.1016/j.ppnp.2016.04.002).

Fluctuations of QCD conserved charges

In many different fields, the study of fluctuations can provide physical insights into the underlying microscopic physics. The fluctuations can become invaluable physical observable in spite of their difficult character. Fluctuations are powerful tools to diagnose microscopic physics, to trace back the history of the system and the nature of its elementary degrees of freedom (see M. Asakawa and M. Kitazawa, Prog. Part. Nucl.Phys. 90, 299 (2016) doi:10.1016/j.ppnp.2016.04.002).

My work is focused on uctuations of conserved charges in the QCD Phase Diagram (B, Q, S) explored through their cumulants.

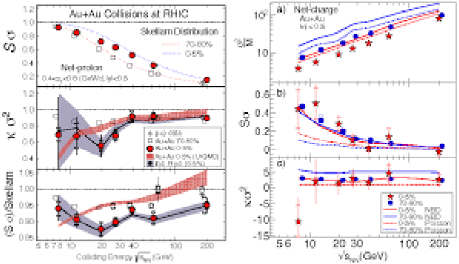


Figure: Combinations of Cumulants in HIC N.R.Sahoo and the Star Collaboration 2014 J.Phys.:Conf. Ser. 535 012007

How can we explore the Phase Diagram?

Experiment

Today for exploring the QCD Phase Diagram,relativistic collisions between heavy ions in LHC at CERN and in RHIC at BNL are used. During these collisions a system is created, where the quarks are no longer confined into hadrons. In these experiments is possible to measure Multiplicity of particles, fluctuations of net charge number, elliptic flow, etc... But the experimental results must be interpreted with the help of some theory

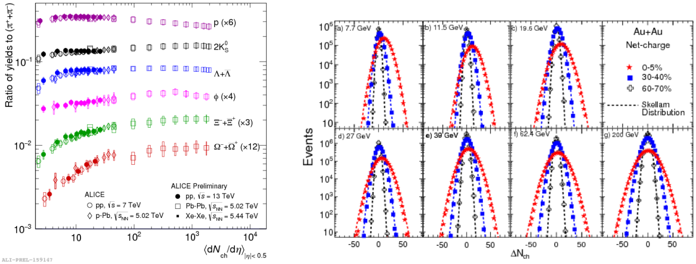


Figure: Particle Multiplicity from Nucl.Phys. A982 (2019) 467-470 and Distribution of Net-Charge at different centrality N.R.Sahoo Star Coll. 2014 J.Phys.:Conf. Ser. 535 012007

How can we explore the Phase Diagram?

Lattice Simulation

Another method for exploring the phase diagram are the Lattice QCD Calculation where the space-time is discretized and it is possible obtain non-perturbative results (e.g. glueball and meson mass). This method is very powerful, it is based on the Montecarlo Algorithm for the calculation of Euclidean path integral.

How can we explore the Phase Diagram?

Lattice Simulation

Another method for exploring the phase diagram are the Lattice QCD Calculation where the space-time is discretized and it is possible obtain non-perturbative results (e.g. glueball and meson mass). This method is very powerful, it is based on the Montecarlo Algorithm for the calculation of Euclidean path integral.

In this approach there appears the sign problem for non-vanishing chemical potential This problem is overcome using a Taylor Expansion of =T at least in the region of Phase Diagram where this procedure is possible

E ective Field Theories

The basic idea of an Effective Field Theories (EFT) is that, if one is interested in describing phenomena occurring at a certain (low) energy scale, one does not need to solve the exact microscopic theory in order to provide useful predictions.

How can we explore the Phase Diagram?

E ective Field Theories

The basic idea of an Effective Field Theories (EFT) is that, if one is interested in describing phenomena occurring at a certain (low) energy scale, one does not need to solve the exact microscopic theory in order to provide useful predictions.

In general, Effective Field Theories are low-energy approximations of more fundamental theories. Instead of solving the underlying theory,low-energy physics is described with a set of variables that are suited to the particular energy region one is interested in.

PNJL

$$L_{PNJL} = \bar{q}(i \quad D \quad \hat{m})q + \frac{1}{2}G[(\bar{q}\sim q)^2 + (\bar{q}i \quad ^5\sim q)^2]+$$
$$+ K f det[\bar{q}(1 + \quad ^5)q] + det[\bar{q}(1 \quad ^5)q] g \quad U(\Phi[A]; \bar{\Phi}[A]; T) \qquad (2)$$

Here $D \quad = @ \quad iA \quad , A \quad = \quad _0 A^0$, the fields Φ and $\bar{\Phi}$ are Polyakov fields defined as:

$$\Phi \quad \frac{1}{N_c} Tr hhL i i \qquad \bar{\Phi} \quad \frac{1}{N_c} Tr hhL^y i i \qquad (3)$$

Where L is the Polyakov loop defined in terms of the gauge field A_4,after Wick rotation:

$$L(\not{x}) \quad P \quad i \, \mathrm{n} \int_0 \mathrm{Z} \, d \quad A_4(\quad ; \not{x}) \, \mathrm{o} \qquad (4)$$

Due to the second term in (2) the PNJL isn't renormalizable and I introduce a cut-off (Λ) for regularizing the integrals.
Below I indicate the quark chiral condensate as:

$$h\bar{q}_i q_i i \quad '_i \quad i = u; d; s \qquad (5)$$

Polyakov Potential

The Polyakov Potential replaces the gluonic interaction of QCD in this EFT. I'm using these two parametrizations:

- Polynomial:

$$\frac{U}{T^4} = \quad \frac{b_2(T)}{2}\bar{\Phi}\Phi \quad \frac{b_3}{6}(\bar{\Phi}^3 + \Phi^3) + \frac{b_4}{4}(\bar{\Phi}\Phi)^2 \qquad (6)$$

$$b_2(T) = a_0 + a_1\frac{T_0}{T} + a_2 \quad \frac{T_0}{T} \quad {}^2 + a_3 \quad \frac{T_0}{T} \quad {}^3$$

- Logarithmic:

$$\frac{U}{T^4} = a(T)\bar{\Phi}\Phi + b(T)\ln\left[1 \quad 6\bar{\Phi}\Phi + 4(\bar{\Phi}^3 + \Phi^3) \quad 3(\bar{\Phi}\Phi)^2\right] \qquad (7)$$

$$a(T) = a_0 + a_1\frac{T_0}{T} + a_2 \quad \frac{T_0}{T} \quad {}^2; \quad b(T) = b_3 \quad \frac{T_0}{T} \quad {}^3$$

The Parameters are fixed for reproducing the lattice data for pure YM-Theory (C.Ratti,et al in Phys.Rev. D 73,014019 (2006) and Nuc.Phys A Vol 814, 1-4 (2008))

Thermodynamics

From PNJL Lagrangian one obtains the Thermodynamic Grand Potential per unit volume($! = \Omega = V$) in Mean Fields Approximation:

$$!(\Phi; \bar{\Phi}; T; M_j; \;_j) = U(\Phi; \bar{\Phi}; T) + G \sum_{i=u;d;s} {'}_i^2 + 4K\, {'}_u\, {'}_d\, {'}_s +$$

$$2 \sum_{i=u;d;s} N_c \int^{\wedge} \frac{d^3p}{(2\;)^3} E_i + T \int^{\wedge} \frac{d^3p}{(2\;)^3} \; z_\Phi^{i+}(E_i; \;_i) + z_\Phi^{i}\;(E_i; \;_i) g \; !$$

Thermodynamics

From PNJL Lagrangian one obtains the Thermodynamic Grand Potential per unit volume($\omega = \Omega/V$) in Mean Fields Approximation:

$$\omega(\Phi;\bar{\Phi};T;M_j;\mu_j) = U(\Phi;\bar{\Phi};T) + G\sum_{i=u;d;s}\varphi_i^2 + 4K\varphi_u\varphi_d\varphi_s +$$

$$2\sum_{i=u;d;s}\left(N_c\int^{\Lambda}\frac{d^3p}{(2\pi)^3}E_i + T\int^{\Lambda}\frac{d^3p}{(2\pi)^3}\{z_\Phi^{i+}(E_i;\mu_i) + z_\Phi^{i-}(E_i;\mu_i)\}\right)$$

From ω it is possible to obtain all thermodynamics quantities of interest:

$$P = -\omega; \quad s = -\frac{\partial\omega}{\partial T}; \quad n_i = -\frac{\partial\omega}{\partial\mu_i}; \quad \chi_n^i = \frac{\partial^n\omega}{\partial\mu_i^n} \tag{8}$$

$$\epsilon = -P + sT + \sum_i \mu_i n_i \tag{9}$$

Cumulants

To describe the fluctuations of conserved charge we can use the cumulants.
The relations between the first four moments and first cumulants read:

Cumulants

To describe the fluctuations of conserved charge we can use the cumulants.
The relations between the first four moments and first cumulants read:

$$
\begin{aligned}
\langle x \rangle_c &= \langle x \rangle \quad M \\
\langle x^2 \rangle_c &= \langle x^2 \rangle \quad \langle x \rangle^2 \qquad {}^2 \\
\langle x^3 \rangle_c &= \langle (x \quad \langle x \rangle)^3 \rangle \qquad {}^3 \\
\langle x^4 \rangle_c &= \langle (x \quad \langle x \rangle)^4 \rangle \quad 3 \;{}^4 \qquad {}^4
\end{aligned}
\tag{10}
$$

Cumulants

To describe the fluctuations of conserved charge we can use the cumulants. The relations between the first four moments and first cumulants read:

$$
\begin{aligned}
hx\,i_c &= hx\,i \quad\quad M \\
hx^2 i_c &= hx^2 i \quad hx\,i^2 \quad\quad ^2 \\
hx^3 i_c &= h(x \quad hx\,i)^3 i \quad\quad ^3 \\
hx^4 i_c &= h(x \quad hx\,i)^4 i \quad 3 \ ^4 \quad\quad ^4
\end{aligned}
\tag{10}
$$

is called kurtosis and is called skewness. This two quantities represent respectively the "sharpness" and asymmetry of the distribution.

Cumulants

To describe the fluctuations of conserved charge we can use the cumulants. The relations between the first four moments and first cumulants read:

$$
\begin{aligned}
hx\,i_c &= hx\,i \quad\quad M \\
hx^2 i_c &= hx^2 i \quad hx\,i^2 \quad\quad {}^2 \\
hx^3 i_c &= h(x \quad hx\,i)^3 i \quad\quad\quad {}^3 \\
hx^4 i_c &= h(x \quad hx\,i)^4 i \quad 3\;{}^4 \quad\quad\quad {}^4
\end{aligned}
\tag{10}
$$

is called kurtosis and is called skewness. This two quantities represent respectively the "sharpness" and asymmetry of the distribution.
The cumulants are more convenient than moments, e.g, when one consider the products of distributions, the cumulant of the product distribution is the product of cumulants.

Cumulants

To describe the fluctuations of conserved charge we can use the cumulants. The relations between the first four moments and first cumulants read:

$$
\begin{aligned}
\langle x \rangle_c &= \langle x \rangle \quad M \\
\langle x^2 \rangle_c &= \langle x^2 \rangle \quad \langle x \rangle^2 \qquad {}^2 \\
\langle x^3 \rangle_c &= \langle (x \quad \langle x \rangle)^3 \rangle \qquad {}^3 \\
\langle x^4 \rangle_c &= \langle (x \quad \langle x \rangle)^4 \rangle \quad 3 \ {}^4 \qquad {}^4
\end{aligned}
\tag{10}
$$

is called kurtosis and is called skewness. This two quantities represent respectively the "sharpness" and asymmetry of the distribution.
The cumulants are more convenient than moments, e.g, when one consider the products of distributions, the cumulant of the product distribution is the product of cumulants.
In my work I consider the following combinations of cumulants:

$$
{}^2 = \frac{\langle x^4 \rangle_c}{\langle x^2 \rangle_c} \qquad \frac{{}^3}{M} = \frac{\langle x^3 \rangle_c}{\langle x \rangle_c} \tag{11}
$$

Results

In the last part of my presentation I show the numerical results obtained with PNJL for the following observables:

- isentropic trajectories
- speed of sound
- fluctuations of net Baryon number

Results

In the last part of my presentation I show the numerical results obtained with PNJL for the following observables:

- isentropic trajectories
- speed of sound
- fluctuations of net Baryon number

From a general point of view the phase diagram of QCD is a 4-dimension space: 3 dimensions for the quark chemical potentials and 1 for temperature.I perform my calculations in the following scenarios:

Results

In the last part of my presentation I show the numerical results obtained with PNJL for the following observables:

- isentropic trajectories
- speed of sound
- fluctuations of net Baryon number

From a general point of view the phase diagram of QCD is a 4-dimension space: 3 dimensions for the quark chemical potentials and 1 for temperature.I perform my calculations in the following scenarios:

- Symmetric chemical potential: ${}_u = {}_d = {}_s = \frac{1}{3}\ {}_B$
- (Quasi-)Neutral Strangeness: ${}_u = {}_d = \frac{1}{3}\ {}_B\ ,\ \ {}_s = 0$
- HIC: $\frac{n_Q}{n_B} = 0{:}4$, $n_s = 0$

Results:isentropic trajectories

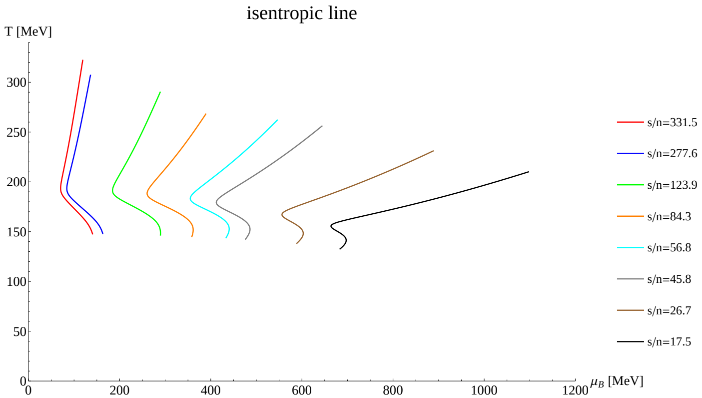


Figure: Isentropic Trajectories in the (Quasi)-neutral strangeness (M.Motta et al in prep.(2019)

Results:speed of sound

$$c_s^2 = \frac{@P}{@} \tag{12}$$

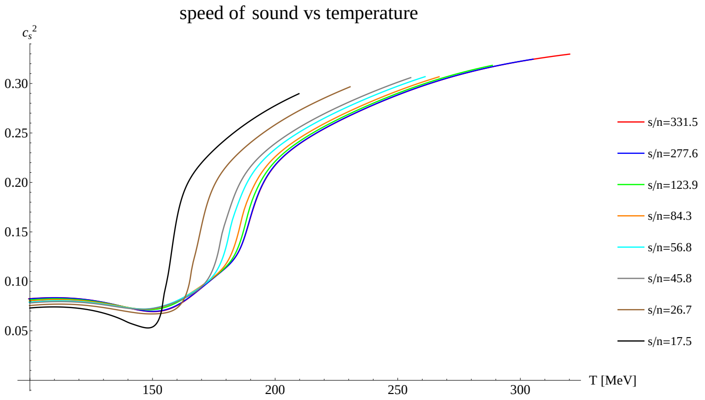


Figure: Speed of sound in the (Quasi)-neutral strangeness on the isentropic trajectories (M.Motta eta al in prep.(2019))

Results:fluctuations in Symmetric scenario

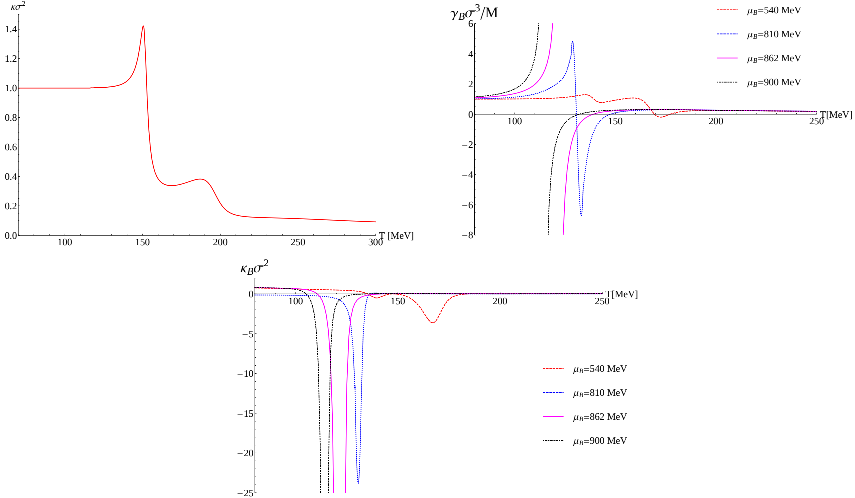


Figure: M.Motta et al [1909.05037]

Results:fluctuations in Neutral strangness scenario [?]

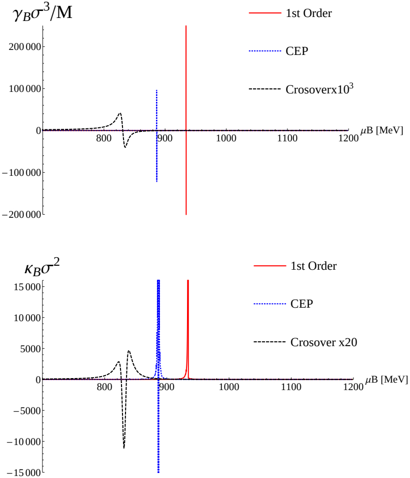


Figure: $(T_1 = 122{:}9; T_2 = 132{:}9 T_3 = 142{:}9)$ MeV,M.Motta et al [1909.05037]

Conclusions and outlook

Conclusions:

- PNJL model provides a good qualitative and semi-quantitative guidance to describe the chiral and deconfinement QCD transition
- Nature of the active degrees of freedom is displayed by high order cumulants (e.g. kurtosis)

outlook:

- Calculation in the HIC scenarios are in progress and other thermodynamic quantities are currently under investigation
- I'm going to perform the calculation of mixed flavours susceptibility for the comparison with Lattice QCD and experimental results.

Thank you for your attention!

The value of Polyakov Fields is connected to the energy for produce a free quark from the vacuum:

$$\Phi \qquad e^{\quad F_q} \tag{13}$$

In the confined region, where it is not possible to create a sigle quarks from vacuum, $F_q \to +\infty$ then $\Phi \to 0$.
In the deconfined region F_q is finite and $\Phi \neq 0$.
At extremely high temperature $\Phi \to 1$. In any case Φ is smaller than unity

Generators of Fermi Functions

In PNJL lagrangian appears the functions z^{i}_{Φ}

$$z^{i+}_{\Phi}(E_i;\ \ _i) \quad \ln[1+N_c(\Phi+\bar{\Phi}\,\mathrm{e}^{\ \ (E_i\ \ \ _i)})\,\mathrm{e}^{\ \ (E_i\ \ \ _i)}+\mathrm{e}^{\ 3\ (E_i\ \ \ _i)}] \tag{14}$$

$$z^{i}_{\Phi}\ (E_i;\ \ _i) \quad \ln[1+N_c(\bar{\Phi}+\Phi\,\mathrm{e}^{\ \ (E_i+\ _i)})\,\mathrm{e}^{\ \ (E_i+\ _i)}+\mathrm{e}^{\ 3\ (E_i+\ _i)}] \tag{15}$$

The derivative of this function on chemical potential $\ _i$ are the Fermi modified function:

$$\begin{aligned} f^{i}_{\Phi}\ (E_i;\ \ _i) \quad & \frac{T}{N_c}\frac{@z^{i}}{@\ \ _i} \\ = & \frac{(\Phi+2\bar{\Phi}\,\mathrm{e}^{\ \ (E_i\ \ \ _i)})\,\mathrm{e}^{\ \ (E_i\ \ \ _i)}+\mathrm{e}^{\ 3\ (E_i\ \ \ _i)}}{1+N_c(\Phi+\bar{\Phi}\,\mathrm{e}^{\ \ (E_i\ \ \ _i)})\,\mathrm{e}^{\ \ (E_i\ \ \ _i)}+\mathrm{e}^{\ 3\ (E_i\ \ \ _i)}} \end{aligned} \tag{16}$$

Mass Gap Equation

The PNJL Lagrangian is chiral symmetric if $m_i = 0$. For non vanishing current mass chiral symmetry is explicitly broken. Moreover, at low temperature and chemical potential, chiral symmetry is also dynamically broken by self-interaction of quarks: the chiral condensate is negative and large.

Mass Gap Equation

The PNJL Lagrangian is chiral symmetric if $m_i = 0$. For non vanishing current mass chiral symmetry is explicitly broken. Moreover, at low temperature and chemical potential, chiral symmetry is also dynamically broken by self-interaction of quarks: the chiral condensate is negative and large. The mass gap equation for the quark of species i reads:

$$M_i = m_i \quad 2G\,\varphi_i \quad 2K\,\varphi_j\,\varphi_k; \quad i \neq j \neq k \tag{17}$$

Mass Gap Equation

The PNJL Lagrangian is chiral symmetric if $m_i = 0$. For non vanishing current mass chiral symmetry is explicitly broken. Moreover, at low temperature and chemical potential, chiral symmetry is also dynamically broken by self-interaction of quarks: the chiral condensate is negative and large. The mass gap equation for the quark of species i reads:

$$M_i = m_i - 2G\varphi_i - 2K\varphi_j\varphi_k; \quad i \neq j \neq k \tag{17}$$

The second term of the RHS of the equation is due to the 4-fermion interaction vertex and the third term is due to the 6-fermion interaction vertex. This vertex mixes the chiral condensates one with an others.