Second Year Seminar

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FACTORIZATION and UNIVERSALITY in e^+e^- processes

1. FACTORIZATION AND PREDICTIVE POWER

Perturbation Theory and Quantum Fields

- Consider a Quantum Field Theory which describes some kind of interaction modulated by a certain coupling g.
- Suppose that we have measured a certain physical observable O (e.g. a cross section).

Is this observable describable by the theory?

1. Analytic computation
2. Comparison with data

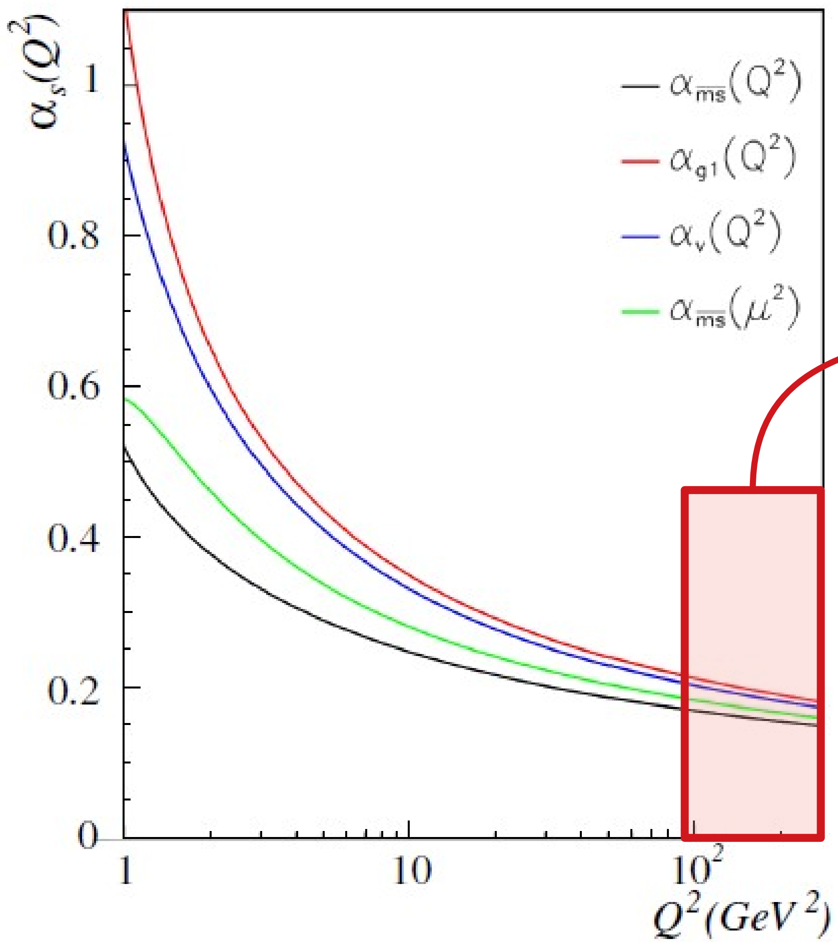
$$O_{\mathrm{meas}} = O^{[0]} + g^2 O^{[1]} + g^4 O^{[2]} + \dots$$

WARNING:
It only works if the coupling can be considered a *small parameter*, suitable for the expansion.

In general, the coupling runs with energy: $$\frac{dg(Q)}{d\log Q^2} = \beta\left(g(Q)\right)$$

Not all the energy scales may be appropriate to expand in powers of g

Lets focus on Quantum ChromoDynamics (**QCD**):



Suitable region to apply perturbation theory

INFRARED PROBLEM: Very dangerous *infrared divergences* appear that make the perturbative expansion unreliable

Asymptotic Freedom in the high energy limit

$$Q \to \infty$$

Coincides with MASSLESS LIMIT

$$m \to 0$$

Infrared Safeness and Predictive Power

Why is this problem so severe in QCD?

Asymptotic states (HADRONS) → They are different! ← Elementary quanta (QUARKS and GLUONS)

CONFINEMENT

Only *very few* observables can be fully described by perturbative QCD.

INFRARED SAFE quantities (very few):

- Inclusive cross sections as e^+e^- in hadrons
- Event shape distributions

$$O_{\text{meas}} \overset{m\to 0}{\sim} \widehat{O} = \widehat{O}^{[0]} + \alpha_S\, \widehat{O}^{[1]} + \dots$$

NOT INFRARED SAFE quantities (the vast majority):

- DIS cross section
- Jet cross sections
- ...

$$O_{\text{meas}} \sim\ ?$$

Factorization

In general, observables are a combination of long- and short-distance contributions.
If we are able *to prove* that they factorize in the massless limit, then:

$$\mathrm{O}_{\mathrm{meas}} \overset{m \to 0}{\sim} \mathcal{H} \times (\mathcal{C} \times \mathcal{S})$$

Short-distance contributions or HARD PART.

- Fully computable within perturbative QCD:
  $\mathcal{H} = \mathcal{H}^{[0]} + \alpha_S\, \mathcal{H}^{[1]} + \ldots$
- Process dependent.
  It is the *signature* of the process, all the kinematics infos are here.

Long-distance contributions or IR PART.

- Infrared divergent at perturbative level.
  It cannot be computed in perturbative QCD.
  → **phenomenology**, lattice...
- Universal. It should not depend on the choice of the process.

The factorization properties save the predictive power of the QCD

2. IR PART AND UNIVERSALITY

IR Part and Universality

The universality of the IR part is crucial for the feasibility of the strategy:

1. Prove that the factorization property holds for the process in exam.
   The proof starts from perturbation theory, graph by graph, and then is extended beyond the perturbative level.

2. Compute the Hard part up to the desidered power of α_s.

3. Extract the IR part from the data of simpler processes (e.g. DIS…) and then use it to predict the cross section of the process in exam.

The IR part is universal ⟷ Color confinement *does not depend* on the specific process.

However, not all the objects that build up the IR part have the same “level” of universality

IR Part and Universality

COLLINEAR FACTOR(S)

- Collinear divergent contributions at perturbative level.
- It is associated to collinear emission of massless particles.
- Their presence is unavoidable for not fully inclusive cross sections, since each collinear factor is associated to a hadron:

  Initial State (incoming hadrons): **PDF**s

  Final State (jets): **FF**s

Hence there is a direct link between collinear factors and data.

SOFT FACTOR

- Soft divergent contributions at perturbative level.
- It is associated to soft emission of massless particles
- It could be non-trivial because of the *kinematics* of the process.

  Trivial: **Collinear Factorization**

  Non-trivial: **TMD Factorization**

- It encodes the *correlation* between collinear factors, hence cannot be extracted independently from them.

This is the Soft Factor Problem.

A Hierarchy of Universality

UNIVERSAL

The Collinear Factors belong to this category.
They are sensitive *only* to their own collinear part variables.
The rest of the process is replaced by a (light-like) Wilson Line in the direction opposite to the reference hadron.

Definition:
Processes with the same number of Collinear Factors (both in initial or in final state) belongs to the same HADRON-CLASS.

HADRON-CLASS UNIVERSAL

The Soft Factor (of the process) belongs to this category.
It is blind to the rest of the process *apart from* the number of collinear parts involved into it, each one appearing as a slightly tilted Wilson Line.

PROCESS DEPENDENT

The Hard Part belongs to this category.
It is totally process dependent, but it can be computed in perturbation theory at the desired accuracy.

3. COLLINEAR VS TMD FACTORIZATION

Collinear Factorization

Real emission of (hard) particles crossed by the final state cut.

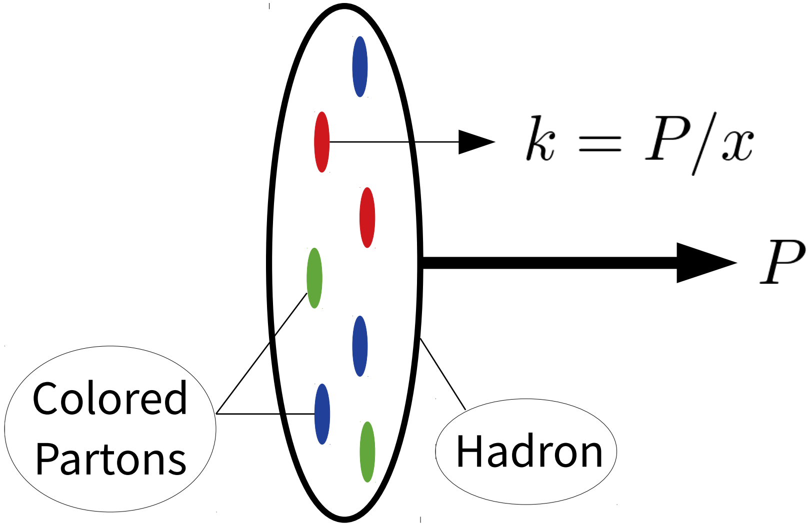
Collinear Factor *not* associated with any hadron.

All the information about the **transverse motion of the partons** confined inside the initial state/detected hadrons is **completely washed out**.

The Soft Factor becomes trivial:

$$\mathcal{S} = 1$$

There is no correlation between Collinear Factors: **1-D** point of view on confinement (pancakes)



This is a well known and widely studied process (LEP…).

The cross section is differential with respect to:

- the CM energy Q ,
- the fractional energy z of the detected hadron. $z = \dfrac{E_H}{Q/2}$

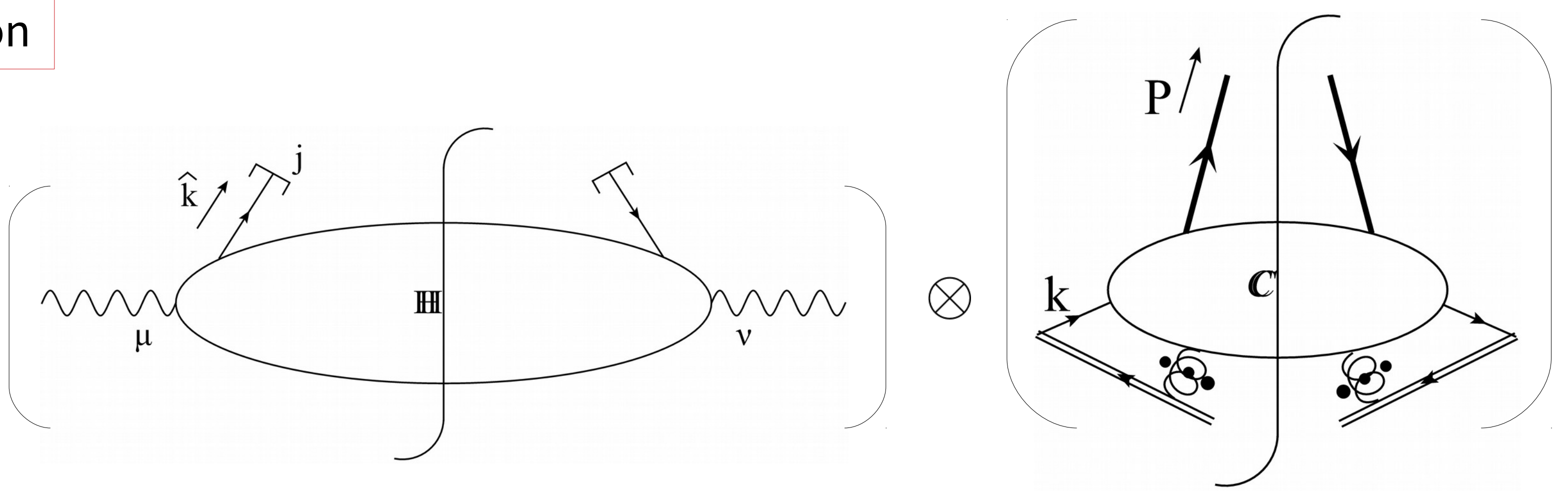
$$\frac{d\sigma}{dQ\,dz} = \sum_j \frac{d\widehat{\sigma}_j}{dQ} \otimes d_{H_j}\,(z,\,Q)$$

Partonic Cross Section

Collinear Fragmentation Function

It describes how a parton of flavor j and collinear momentum fraction $z/\widehat{z}$ fragments into the hadron H.

$W^{\mu\,\nu} =$



TMD Factorization

Real emissions of (hard) particles are not present.

Collinear Factors are always associated with a hadron.

The information about the **transverse motion of the partons** confined inside the initial state/detected hadrons **survives**.

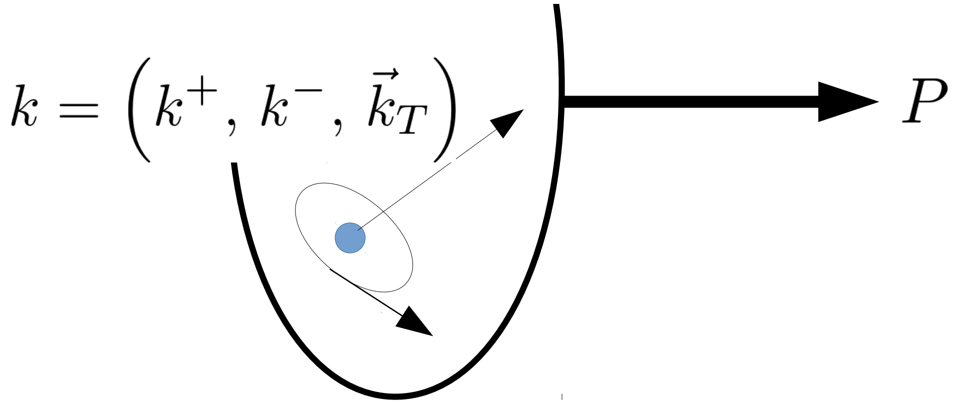
Non-trivial subtractions of the double counted soft-collinear contributions.

The Soft Factor is highly non-trivial:

$$\mathcal{S} = \mathcal{S}\left(k_{S,\,T};\ \mu;\ \{y_1,\ y_2 \ldots y_N\}\right)$$

Introducing rapidity cut-offs into Collinear Factors to regularize their rapidity divergences

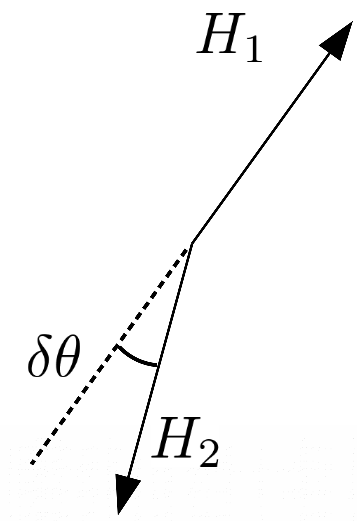
The Collinear Factors are correlated:
3-D point of view on confinement



$e^+e^- \to H_1 H_2$ back-to-back: TMD Factorization

The cross section is differential with respect to:

- The CM energy Q;
- The fractional energies of the two hadrons,
- The transverse momentum of the virtual photon q_T , which is directly related to the small angle shift δθ between the hadrons.



$$\frac{d\sigma}{dQ\, dz_1\, dz_2\, dq_T} = \sum_j \mathcal{H}_{j,\bar{j}} \int \frac{d^2\vec{b}_T}{(2\pi)^2}\, e^{i\vec{q}_T\cdot\vec{b}_T} \times$$
$$\times\, D_{H_1/j}\left(b_T,\, z_1,\, y_1\right) D_{H_2/\bar{j}}\left(b_T,\, z_2,\, y_2\right) \mathcal{S}\left(b_T,\, y1 - y_2\right)$$

Hard Part

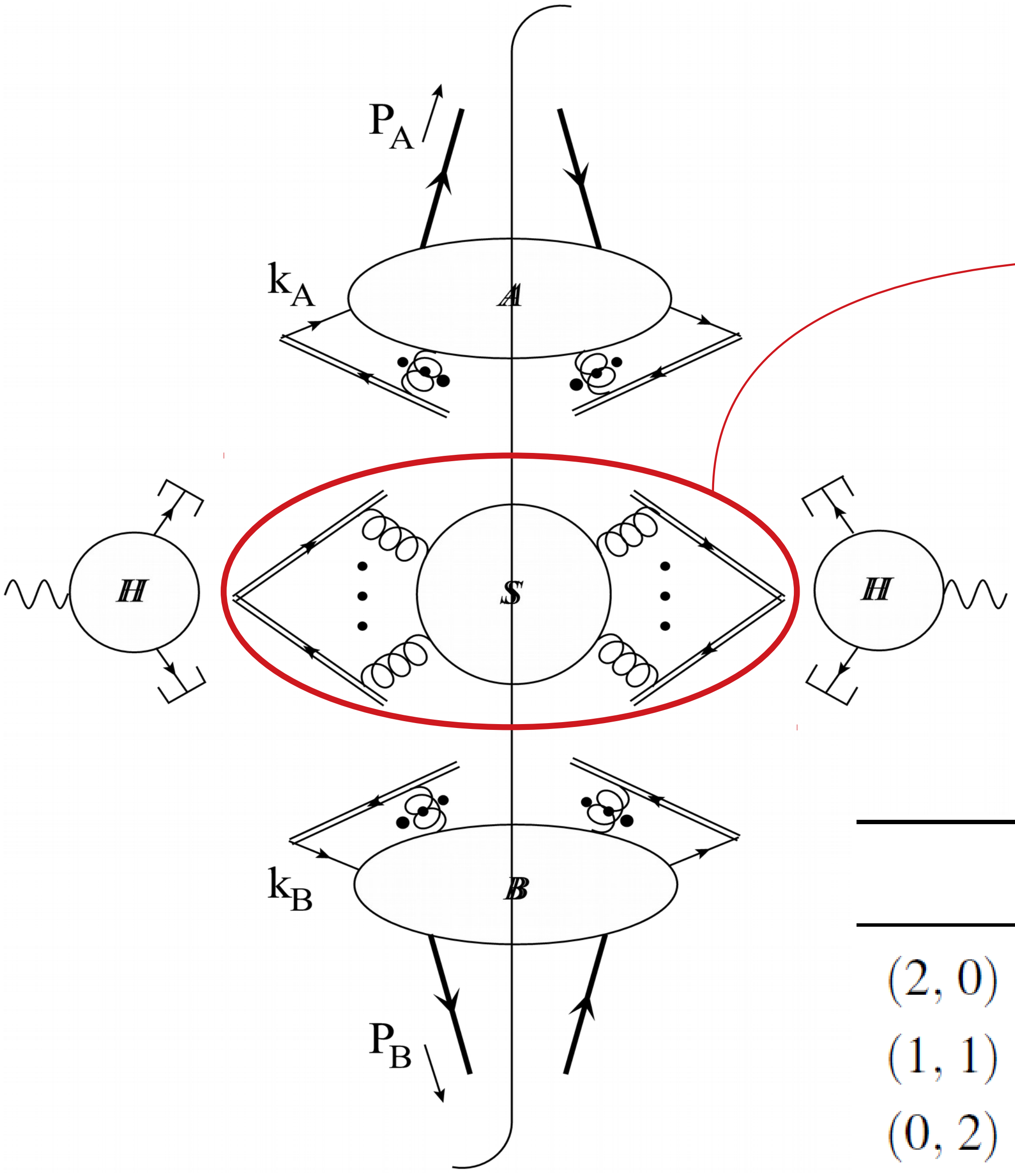
Transverse Momentum Dependent (TMD) Fragmentation Functions

They describe how a parton of flavor j , collinear momentum fraction z , and transverse momentum $\vec{k}_T$ fragments into the hadron H.

Non-trivial 2-h Soft Factor

Notice that all dependences on the rapidity cut-offs y_1 and y_2 disappears in the full cross section.

The Square Root Definition



SOFT CORRELATION:
The two Collinear Factors can exchange information *only* through soft gluons

Property of 2-h class:
The soft factor can be reabsorbed into a "new"definition of the TMDs.

2-hadron class	
(2, 0)	Drell-Yan
(1, 1)	SIDIS
(0, 2)	$e^+e^- \to H_A\, H_B\, X$

The Square Root Definition

Naively we assign a square root of the soft factor to each of the TMDs in the cross section, which now is "Parton Model-like":

$$\frac{d\sigma}{dQ\, dz_1\, dz_2\, dq_T} = \sum_j \mathcal{H}_{j,\bar{j}} \int \frac{d^2\vec{b}_T}{(2\pi)^2}\, e^{i\vec{q}_T\cdot\vec{b}_T}\, D^{\text{sqrt}}_{H_1/j}(b_T,\, z_1,\, y_n)\, D^{\text{sqrt}}_{H_2/\bar{j}}(b_T,\, z_2,\, y_n)$$

This re-defiinition of TMDs solves the Soft Factor problem in the 2-h class!
Furthermore, it has a lot of advantages (compatibility with factorization, only one left rapidity cut-off, easier perturbative computations, gauge invariance is more explicit….) ✓

...however it reduces the universality of the TMDs!
The new TMDs are universal *only* inside the 2-h class. X

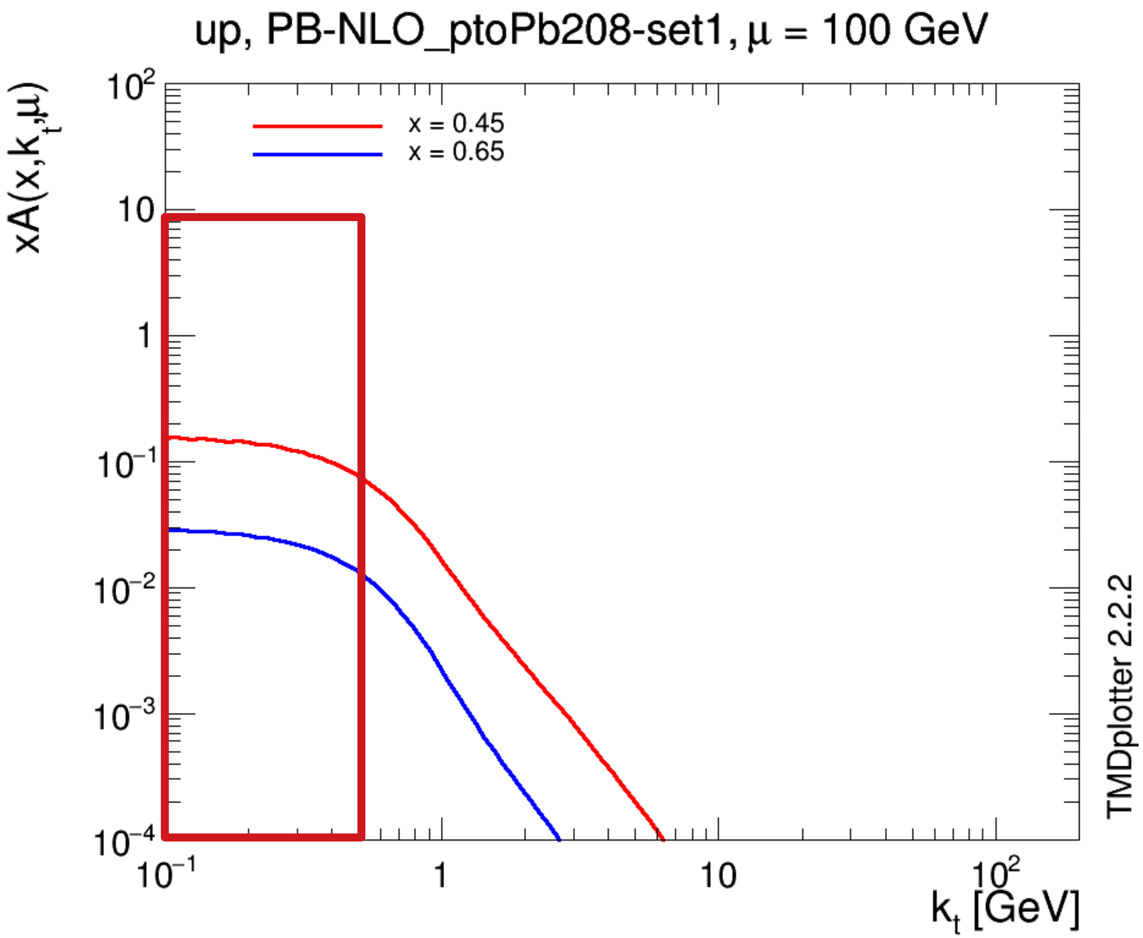
→ The square Root Definition is optimal in the 2-h class,
but it becomes problematic if one has to deal with N-h class processes, where N ≠ 2

The Square Root Definition: Comparison

Background:
The factorization procedure can be applied to the Collinear Factors and Soft Factor themselves and separate out the truly non-perturbative part from what is still computable in perturbative QCD.

For TMDs and Soft Factor:

- Perturbative part is at *large values* of k_T ,
- Non-perturbative part (MODEL) at *low values of* k_T .



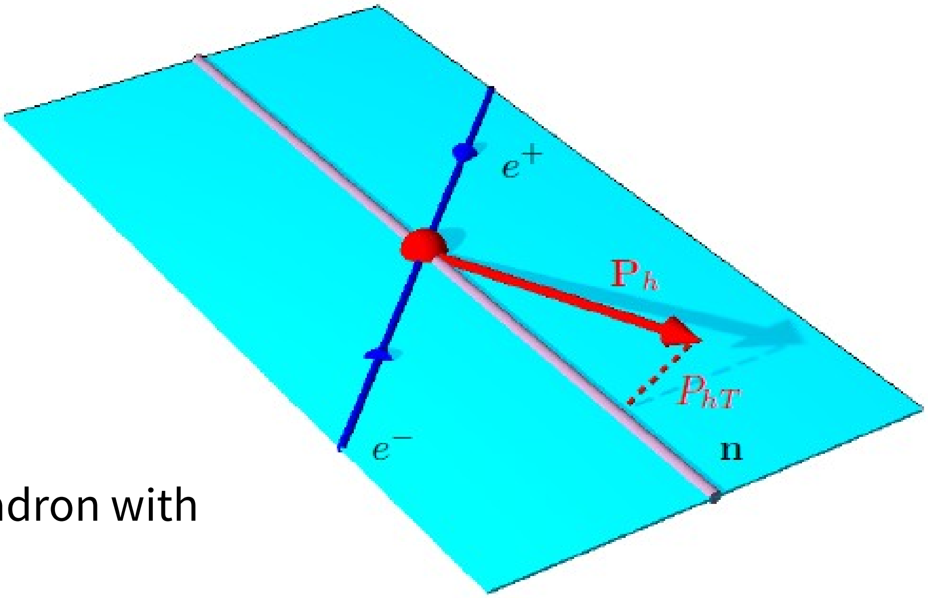
The comparison between the two definitions shows that they differ *only* in the non-perturbative part by a square root of the Soft Model

$$\widetilde{D}^{\text{sqrt}}_{H/j}\Big(z,\, b_T;\, \mu,\, y_n\Big) = \\ = \widetilde{D}_{H/j}\Big(z,\, b_T;\, \mu,\, y_n\Big) \times \sqrt{M_S(b_T)}$$

4. Beyond 2-hadron class

Outside the 2-h class: Belle Data

The considered process is again e^+e^- in one hadron, $e^+e^- \to HX$, but this time the Belle Collaboration provides a cross section differential in more variables:



- The CM energy Q,
- The fractional energy z of the detected hadron,
- The thrust axis and the value of the thrust T,
- The transverse momentum P_T of the detected hadron with respect to the thrust axis (NEW!)

- There is always *at least* one real emission. → Collinear Factorization
- The measure of P_T gives a direct probe of the parton's transverse motion → TMD Fragmentation

?

Factorization of the Cross Section

Remember:

$$T = \max_{\vec{n}} \frac{\sum_i |\vec{P}_i \cdot \vec{n}|}{\sum_i |\vec{P}_i|}$$

Assuming that the thrust axis exactly coincides with the axis of the jet, the cross section can be written as:

$$\frac{d\sigma}{dQ\, dz\, dT\, dP_T^2} = \sum_j \int \frac{d^2\vec{b}_T}{(2\pi)^2}\, e^{i\vec{P}_T\cdot\vec{b}_T}\, \frac{d\widehat{\sigma}_j}{dQ\, dT} \otimes D_{H/j}\left(zb_T,\, z,\, y_1\right)$$

The cross section seems to depend on the choice of the rapidity cut-off

The thrust dependence goes into the partonic cross section.

The transverse momentum dependence goes into the TMD FF, given by the original definition (not the square root!).

Rapidity Dilations

Actually, TMDs have a hidden symmetry.
They are invariant with respect to the choice of the rapidity cut-off y_1 if the following transformations are performed *simultaneously*:

- The rapidity cut-off is shifted.

$$y_1 \mapsto y_1' = y_1 - \theta$$

- The model (non-perturbative part) is *dilated* by a factor which is fully computable in perturbative QCD.

$$M_j\left(b_T\right) \mapsto M_j'\left(b_T\right) = e^{-\frac{1}{2}\theta \widetilde{K}\left(b_T\right)}\, M_j\left(b_T\right)$$

I called the previous set of transformations Rapidity Dilations.
Under their action the TMDs are invariant:

$$D_{H/j}\left(z,\, b_T,\, y_1\right) \mapsto D_{H/j}'\left(z,\, b_T,\, y_1'\right) = D_{H/j}\left(z,\, b_T,\, y_1\right)$$

The TMDs are well defined *modulo a rapidity dilation*. ⟷ Similar to a gauge transformation

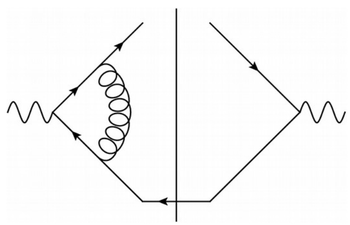
Perturbative computation at 1 loop

I computed the cross section at perturbative level, at 1 loop.
Defining $\tau = 1 - T$, I found in the almost 2-jet limit ($\tau \sim 0$):

$$d\sigma^{[1]}(\tau,\ b_T,\ z,\ \mu/Q) = d\widehat{\sigma}^{[1]}(\tau,\ z,\ \mu/Q) + d\widehat{\sigma}^{[0]} \times D^{[1]}(\epsilon,\ z,\ \mu/Q,\ \bar{y}_1)\ \delta(\tau)$$

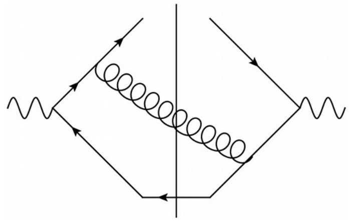
Computation of the following Feynman diagrams in the almost 2-jet limit:

Virtual:

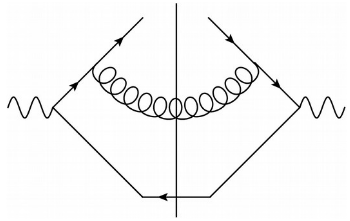


+ h.c.

Real:



+ h.c.



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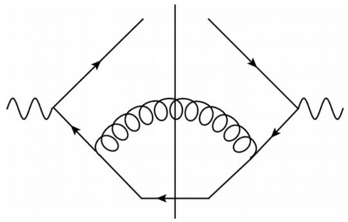

- ϵ :Explicit (regulated) collinear divergence at perturbative level.
- y_1: Peculiar choice of the rapidity cut-off, which depends on the thrust.
- $\delta(\tau)$: The TMDs is sensitive only to the jet direction, hence the thrust dependence is trivial.

5. Conclusions

Summarizing...

1. The Factorization property of the cross sections saves the predictive power of the QCD. It allows to separate what is computable in perturbation theory (HARD PART) from what has to be extracted from experimental data (IR PART).

2. The IR part can be expressed as a product of Collinear Factors and eventually a Soft Factor. Only the first are truly universal objects, since the Soft Factor depends on the overall number of Collinear Factors of the full process (n-hadron class universal).

3. Transverse Momentum Dependent (TMD) factorization is the most complex issue. The accepted definition for the TMDs (square root definition) is actually less universal than expected and can only be used inside the 2-h class.

4. The new data from Belle Collaboration offers the possibility to investigate the TMDs outside the 2-h class. This forces us to modify the definition of the TMDs and to introduce a new symmetry: rapidity dilations.

5. Future phenomenological analyses will be devoted to extract the truly universal TMDs. Once they will be known, we could use them to predict the cross sections of a very wide range of processes.

THANK YOU FOR YOUR ATTENTION!