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University of Turin - Physics Department - 2nd year seminar

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Research Field: Cosmological Singularities in String Theory

Based on "*On the Origin of Divergences in Time-Dependent Orbifolds*"
(*Eur. Phys. J. C 80, no.5, 476 (2020)* - A.A., R. Finotello, I. Pesando) and ongoing work

Where General Relativity fails...

Einstein GR predicts the existence of singular points where causal geodesics end within finite eigentime

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SPACETIME SINGULARITIES
(Big bang singularity and Black holes)

GR loses its predictive value because the high curvature and energies involved are near the Planck scale

…a theory of Quantum Gravity is needed!

The best developed candidate is

STRING THEORY

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Point-like particles of particle physics are replaced by one-dimensional objects

Why String Theory?

- ST can describe the exchange of gravitons, which are the quanta of the gravitational force

- GR can be obtained as the (classical) low-energy limit of the gravitational interaction in ST

- ST offers a framework capable of incorporating also the other fundamental interactions of the standard model (electromagnetic, weak and strong)

- But that's not all…

Because String Theory already successfully resolves important classes of singularities!

A path to follow

Roughly speaking, singularities can be *TIMELIKE* or *SPACELIKE*

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String Theory has proven capable of giving a satisfactory description of *TIMELIKE STATIC SINGULARITIES*

How? Through the introduction of additional degrees of freedom
(twisted closed strings and wrapped D-branes)

[*Big Bang Models in String Theory - B.Craps*]

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But cosmological singularities are *SPACELIKE* and need a *TIME-DEPENDENT* background!

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Where and how can we deal with them in String Theory?

Orbifolds: very useful toy models

An orbifold O is a generalization of a manifold M where some points are identified under the action of a group G of its isometries such that O can be thought of as M/G

Fixed points on G ⟺ Singular points on O

<u>Two trivial examples: the circle and the 2D cone</u>

- ❑ $\mathbb{R}/\mathbb{Z}$ $\quad x \sim x + 2\pi n R, \;\; n \in \mathbb{Z}$ — No singular points
- ❑ $\mathbb{R}^2/\mathbb{Z}_2$ $\quad (x_1, x_2) \sim (-x_1, -x_2)$ — $(0,0)$ is a singular point
  - It's a **time-independent** orbifold with a **static singularity**
  - In QFT it's singular but in ST it's smooth thanks to the addition of new degrees of freedom

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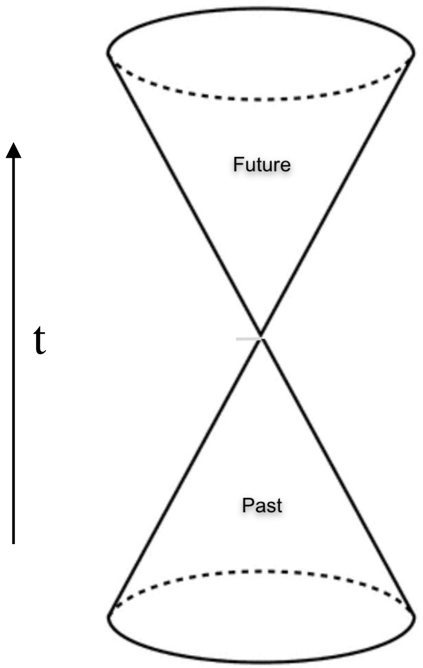
We need a **time-dependent orbifold** since a cosmological singularity is present in a certain region of (or in the whole) space at a specific value of the time coordinate, i.e. it appears in time and then eventually disappears

Minkowski time-dependent orbifolds



We follow a standard orbifold construction procedure for $\mathbb{R}^{1,D-1}$

1. Choose a Killing vector field k of $\mathbb{R}^{1,D-1}$
2. Identify points along the orbits of k such that $P \sim e^{nk}P$ with $n \in \mathbb{Z}$

What do we find?

[*Time dependent orbifolds and string cosmology – L.Cornalba, M.S.Costa*]

- The resulting orbifolds are geometrically different according to the k chosen for the identification
- They can be of four kinds: *boost orbifold (BO)*, *shifted-boost orbifold (SBO)*, *O-plane orbifold (OPO)* and *null-boost orbifold (NBO)*
- They are all singular time-dependent orbifolds **suitable for the study of a singularity like the big bang**

NBO Identifications

$$x = \begin{pmatrix} x^- \\ x^2 \\ x^+ \\ \vec{x} \end{pmatrix} \equiv \mathcal{K}^n x = \begin{pmatrix} x^- \\ x^2 + n(2\pi\Delta)x^- \\ x^+ + n(2\pi\Delta)x^2 + \frac{1}{2}n^2(2\pi\Delta)^2 x^-, \\ \vec{x} \end{pmatrix}$$

The big question

Is the situation as “easy” as in the static case?

The feared answer

Unfortunately, not.

A big problem to deal with

Is the propagation of single particles under control? It seems so

[*Quantum evolution across singularities: the case of geometrical resolutions – B.Craps, F.De Roo, O.Evnin*]

What about the interaction between particles? Things get much worse

[*Instability of spacelike and null orbifold singularities - G. T. Horowitz and J. Polchinski*]

Where is the analytical problem? **Tree level amplitudes**

[*Comments on cosmological singularities in string theory - M.Berkooz, B.Craps, D.Kutasov, G.Rajesh*]

The orbifold four point amplitude $\mathcal{A}$ of closed string tachyons exhibits unusual divergences when we consider the regime of high-energy small-angle scattering (Regge limit)

$$\boxed{\mathcal{A}_{4T}^{closed} \sim \int^{q\sim\infty} \frac{\mathrm{d}q}{|q|}\, q^{\alpha'\left(\frac{4}{\alpha'} - \vec{p}^{\,2}_{\perp t}\right)}}$$

An unsatisfactory explanation

This strange phenomenon has been interpreted in the closed string theory as:

> *"the EXCHANGE OF A GRAVITON of spin J=2 whose coupling with the stress tensor leads to a strong gravitational back-reaction because of the fast oscillations of wave functions near the singularity."*
>
> [*Strings in a time–dependent orbifold - H.Liu, G.Moore, N.Seiberg*]

This may seem reasonable since the graviton is a physical state of closed string theory…

…but we found out that the same problem emerges in the open string theory scattering amplitudes!

$$\boxed{\mathcal{A}_{4T}^{open} \sim \int^{q\sim\infty} \frac{\mathrm{d}q}{|q|}\, q^{\alpha'\left(\frac{1}{\alpha'}-\vec{p}_{\perp t}^{\,2}\right)}\, \mathrm{tr}\left(\{T_1,T_2\}\{T_3,T_4\}\right)}$$

What are we missing?

Going back to QFT

It's easy to show that in a scalar QED theory on the NBO the behaviour of the eigenfunctions of the scalar d'Alembertian near the singularity $u = 0$ reads

$$\phi(u) \propto \frac{1}{\sqrt{|u|}} e^{i\frac{A}{u}}$$

The product of N eigenfunctions generates a singularity $|u|^{-N/2}$ which is technically not integrable.
The exponential term would allow for an interpretation as a distribution provided that $A = 0$ is not an isolated point.
Unfortunately this is exactly what can happen along the compact direction of the NBO, where $A \sim l^2$, with l being the associated discrete momentum.

Fourth and higher order **contact terms are ill defined** because of the presence of a **discrete zero eigenvalue** of the Laplacian of a subspace with vanishing volume

It turns out that the real problem behind these divergences seems to lie in the **non existence of a well defined underlying effective field theory!**

From the String Theory point of view, contact terms may arise from the exchange of **string massive states**: the open string three point amplitude with two tachyons and the first massive state is divergent when some physical polarization are chosen (with all discrete momenta $l_i = 0$)

$$\mathcal{A}^{open}_{TTM} \propto \int_{u\sim 0} du \, \frac{1}{|u|^{5/2}} \, \mathrm{tr}\left(\{T_{(1)}, T_{(2)}\} T_{(3)}\right)$$

The big question: is it possible to cure these divergences? And if yes, how?

- **Generalized Null Boost Orbifold**

$$\phi(u) \propto \frac{1}{\sqrt{|u|}} e^{i\frac{A}{u}}$$

with A being a linear combination of the discrete momentum l and of a continuous momentum p , i.e. $\boldsymbol{A \sim (l \pm p)^2}$, thanks to a **second Killing vector** acting as generator: this avoids the existence of isolated zeros and makes plausible the interpretation of a generic N-point function as a distribution

- **Noncommutative QFT and Twisted Closed Strings**

Our ongoing study seems to suggest that amplitudes on time-dependent orbifolds behave better in a QFT **noncommutative framework**. This could be related to the hypothesis regarding closed strings **winding modes** along the compact direction z, which should contribute to physical observables since the lightest become **massless** near the singularity and enter in the effective action of the low energy description

- **Lightcone String Field Theory**

A second quantization approach may be the best one to shed light on the whole puzzle of string motion in **time-dependent backgrounds**

References

- Gary T. Horowitz and Alan R. Steif - *Singular String Solutions with Nonsingular Initial Data* - Phys. Lett. B, 258:91–96, 1991
- Ben Craps - *Big Bang Models in String Theory* - Class. Quant. Grav., 23:S849–S881, 2006
- Lorenzo Cornalba and Miguel S. Costa - *Time-dependent Orbifolds and String Cosmology* - Fortsch. Phys., 52:145–199, 2004
- Micha Berkooz, Ben Craps, David Kutasov and Govindan Rajesh - *Comments on Cosmological Singularities in String Theory* - JHEP, 03:031, 2003
- Hong Liu, Gregory W. Moore and Nathan Seiberg - *Strings in a Time-Dependent Orbifold* - JHEP, 06:045, 2002
- Hong Liu, Gregory W. Moore and Nathan Seiberg - *Strings in Time-Dependent Orbifolds* - JHEP, 10:031, 2002
- Bruno Durin, Boris Pioline - *Closed Strings in Misner Space: a Toy Model for a Big Bounce?* - NATO Science Series II, 2005

Thank you
for your attention!