CHAOS AND TURBULENCE IN COMPLEX FLUIDS

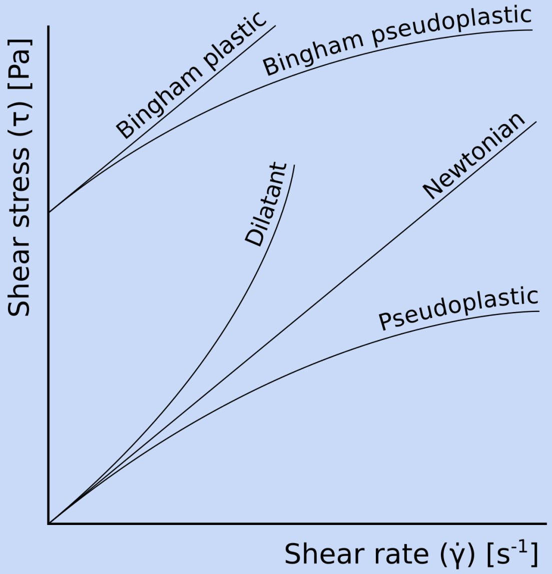
Student:
Leonardo Puggioni

Supervisor:
Stefano Musacchio

Complex Fluids

Complex fluid: coexistence between different phases

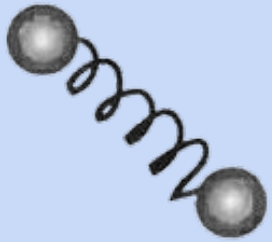
Non-Newtonian rheology: not constant viscosity



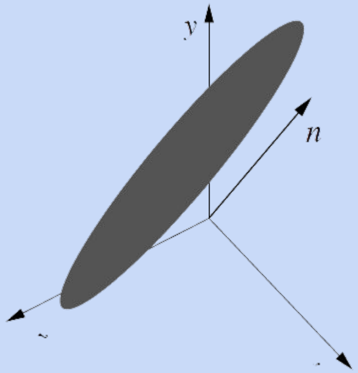
Polymer solutions

Two different macro categories:

1) Elastic polymers (Oldroyd-B, FENE-P), with a viscoelastic stress
2) Rigid rod-like polymers (Doi-Edwards), with a nonlinear viscous stress



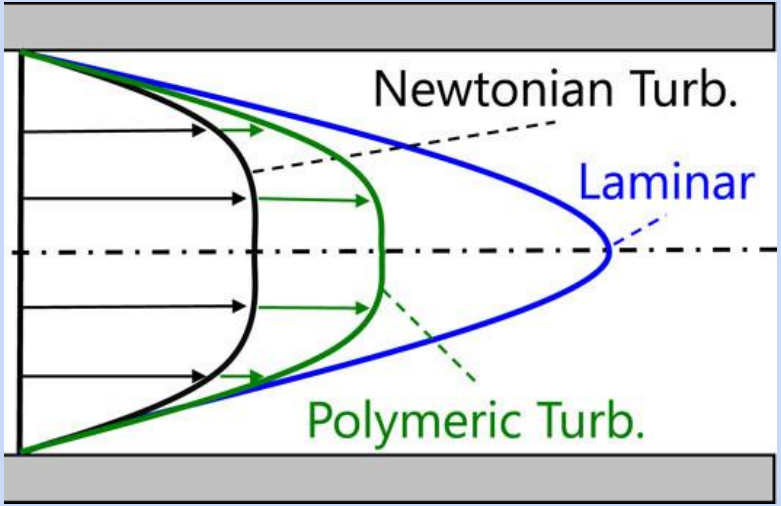
Dumbell model for elastic polymers



Ellipsoidal rod model for rigid polymers

Elastic polymer solutions: high Reynolds

- Turbulent drag reduction at high Reynolds number (1949)

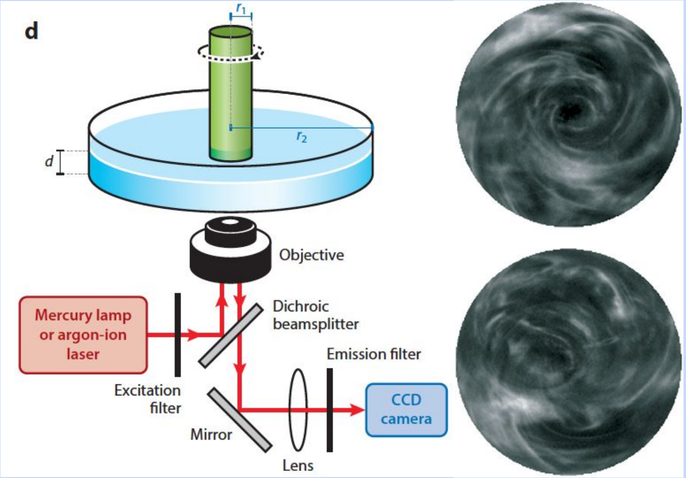


Li Xi (2019)

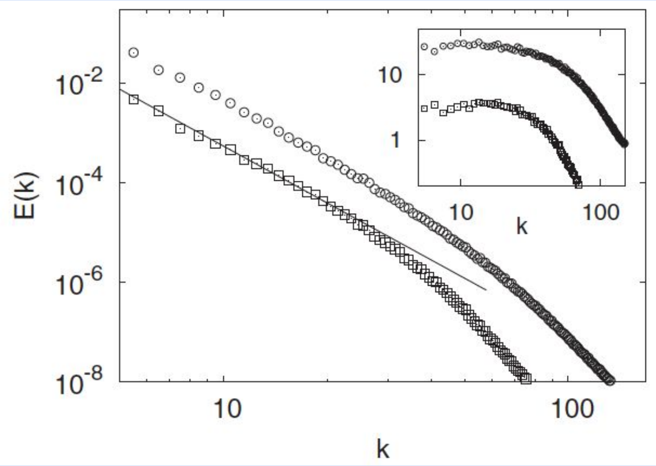
Trans-Alaska pipeline

Elastic polymer solutions: low Reynolds

- Elastic turbulence at low Reynolds number (Groisman & Steinberg, 2000)



First experimental observation
Groisman & Steinberg (2000)



Kinetic energy spectra
Berti, Bistagnino et al. (2008)

Doi-Edwards model

Two-fields description

$$\partial_t u_i + u_k \partial_k u_i = -\partial_i p + \nu \partial^2 u_i - \partial_k \sigma_{ik} + f_i \,,$$

$$\partial_t R_{ij} + u_k \partial_k R_{ij} = (\partial_k u_i)\, R_{kj} + R_{ik}\, (\partial_k u_j) - 2R_{ij}\, (\partial_l u_k)\, R_{kl} \,.$$

Rods dynamics from Jeffery equation (slender rod limit):

$$R_{ij} = \langle n_i n_j \rangle, \qquad \text{with} \qquad \dot{n}_i = (\delta_{ik} - n_i n_k)\,(\partial_l u_k)\, n_l \,.$$

Non-Newtonian polymer stress tensor (ν viscosity and η concentration) and incompressibility condition:

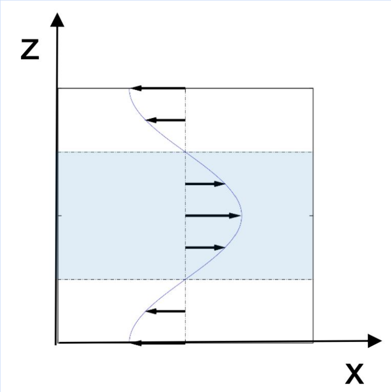
$$\sigma_{ij} = 6\eta_p \nu R_{ij}\, (\partial_l u_k)\, R_{kl} \,, \qquad \partial_k u_k = 0.$$

Kolmogorov flow

Idealized setting of a channel flow without boundaries (Arnold & Meshalkin, 1960, from a Kolmogorov seminar, 1958-1959)

МАТЕМАТИЧЕСКАЯ ЖИЗНЬ В СССР

СЕМИНАР А. Н. КОЛМОГОРОВА ПО ИЗБРАННЫМ ВОПРОСАМ АНАЛИЗА (1958/59 г.)



$$\mathbf{f} = (F\cos(Kz), 0, 0) \quad \Rightarrow \quad \mathbf{u} = (U_0\cos(Kz), 0, 0), \quad U_0 = \frac{F}{\nu K^2}.$$

Stability of laminar flow when (Meshalkin and Sinai, 1961):

$$Re = \frac{U}{\nu K} \leq \sqrt{2}.$$

Kolmogorov flow

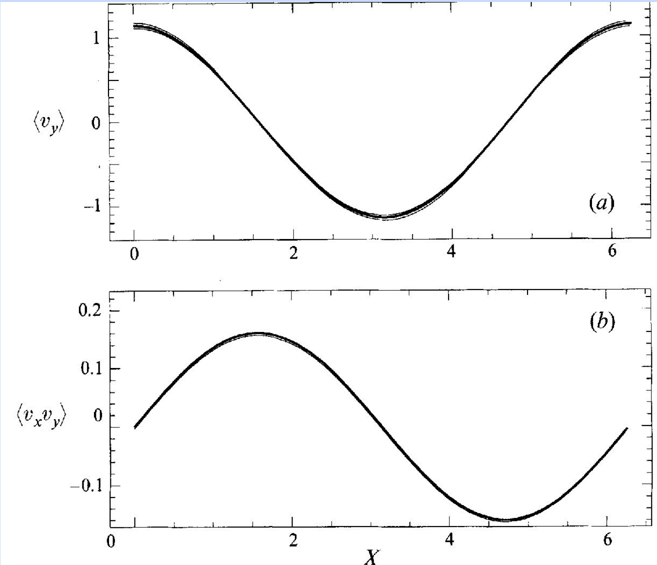
Important feature: symmetry of the laminar flow is statistically present also in turbulent regime ($Re > 10^2$)

$$\mathbf{u}(x,y,z) = \langle \mathbf{u} \rangle (z) + \mathbf{u}'(x,y,z)$$

$$\langle u_x \rangle = U \cos(Kz), \quad \langle u_x u_z \rangle = S \sin(Kz).$$

with $U < U_0$

Borue & Orszag, 1995



Kolmogorov flow

Viscous stress and Reynolds (inertia) stress:

$$\Pi_\nu = \nu \partial_z \langle u_x \rangle = -\nu U K \sin(Kz), \qquad \Pi_r = \langle u_x u_z \rangle = S \sin(Kz).$$

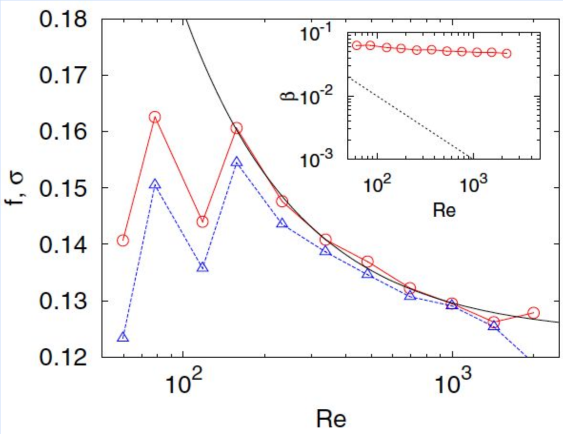
Momentum budget:

$$\partial_z \Pi_r = \partial_z \Pi_\nu + f_z, \quad \Rightarrow \quad F = KS + \nu U K^2.$$

Adimensionalization: friction and stress coefficient

$$f = \frac{F}{KU^2}, \quad \sigma = \frac{S}{U^2} \quad \Rightarrow \quad f = \sigma + \frac{1}{Re}.$$

Asymptotic behaviour: $f = f_0 + \frac{b}{Re}, \quad \sigma = f_0 + \frac{b-1}{Re}.$



Boffetta & Musacchio, 2014

Linear stability

Perturbative multiple-scale analysis (transverse case):

$$\partial_x \to \epsilon \partial_X \,, \quad \partial_y \to \partial_y \,, \quad \partial_t \to \epsilon^2 \partial_T \,,$$

$$u' = u_0 \left(X, y, T\right) + \epsilon u_1 \left(X, y, T\right) + \epsilon^2 u_2 \left(X, y, T\right) ,$$

$$R' = R_0 \left(X, y, T\right) + \epsilon R_1 \left(X, y, T\right) + \epsilon^2 R_2 \left(X, y, T\right) .$$

Main result: rod-like polymers destabilize the flow:

$$Re_c(\eta_p) = \sqrt{2}\sqrt{1 - 6\eta_p \ln 2} < Re_c(0) = \sqrt{2}.$$

Numerical simulations

Parallel code (MPI) with:

- 256^3 periodic domain
- Pseudo-spectral algorithm (linear operators in Fourier space, products in real space)
- 4^{th} order Runge-Kutta scheme for time integration
- Implicit integration of linear terms
- Artificial diffusivity for the polymer field

High-Reynolds flow

Re = 340 (fully developed turbulence):

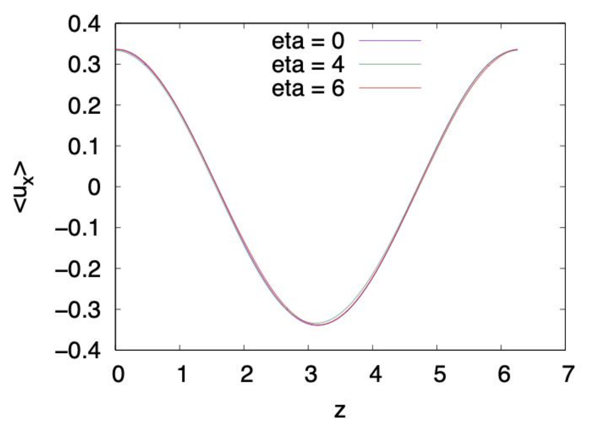
Velocity fluctuations suppressed, but no evidence of drag reduction in Kolmogorov flow with rod-like polymers (contrary to viscoelastic one).

Momentum budget with polymer stress:

$$\Pi_\nu = \nu \partial_z \langle u_x \rangle = -\nu U K \sin(Kz),$$

$$\Pi_r = \langle u_x u_z \rangle = S \sin(Kz),$$

$$\Pi_p = \langle \sigma_{xz} \rangle = -\Sigma \sin(Kz),$$

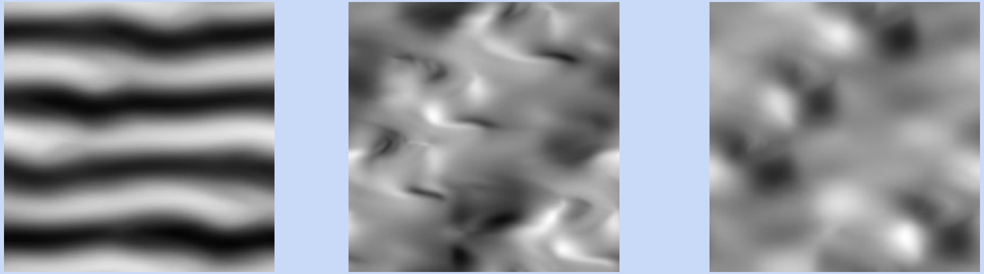


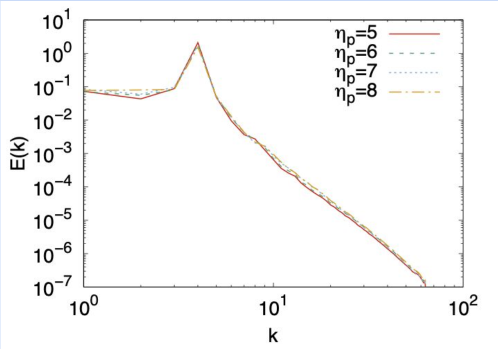
$$\partial_z \Pi_r = \partial_z (\Pi_\nu + \Pi_p) + f_z, \quad \Rightarrow \quad F = SK + \nu U K^2 + \Sigma K.$$

Low-Reynolds flow: chaotic motion

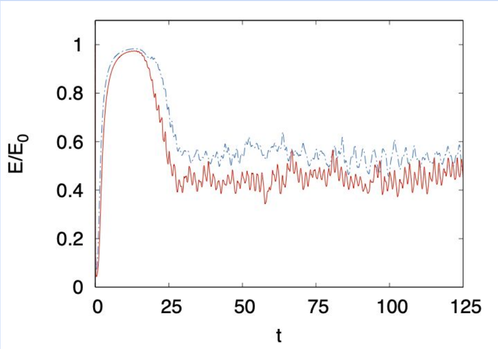
Re = 1: arising of chaotic motion

Snapshots of the three components of the velocity field (η_p = 7)





(left) Kinetic energy spectra. (right) Kinetic energy temporal trends (red: η_p = 8, blue: η_p = 6)



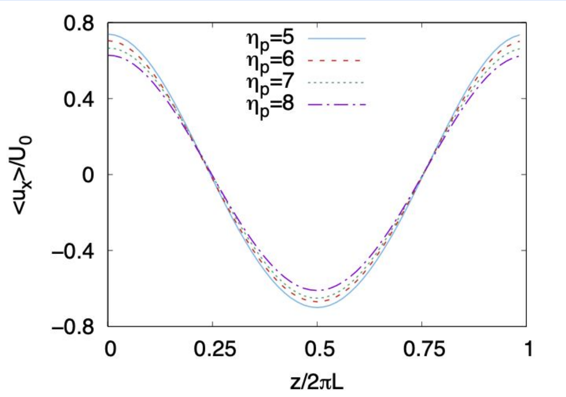
Low-Reynolds flow: drag enhancement

Velocity profile significantly lower than the laminar one:

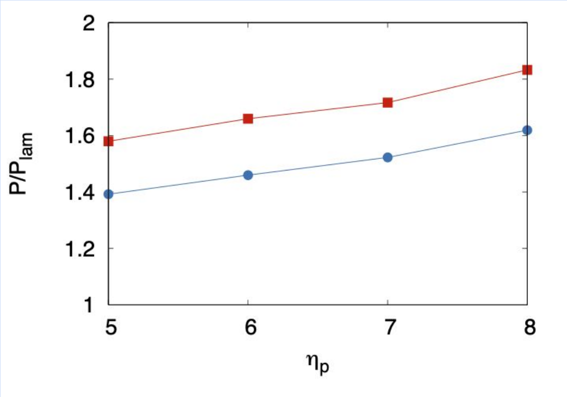
$$\langle u_x \rangle = U \cos(Kz), \qquad U < U_0 .$$

Normalized mean injected power:

$$P = \langle \mathbf{f} \cdot \mathbf{u} \rangle, \qquad \frac{P}{P_{lam}} = \frac{FU/2}{\nu U^2 K^2/2} = \frac{F}{\nu U K^2} .$$



(left) Mean velocity profiles (right) Normalized injected power



Low-Reynolds flow: momentum budget

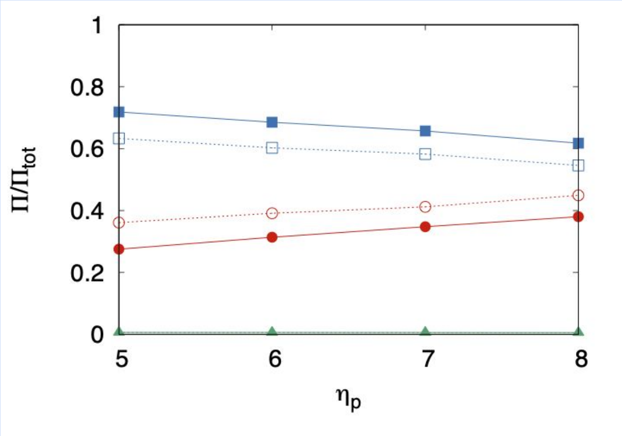
Viscous (Newtonian), Reynolds and polymer stress

$$\Pi_\nu = \nu \partial_z \langle u_x \rangle = -\nu U K \sin(Kz), \qquad \Pi_r = \langle u_x u_z \rangle = S \sin(Kz), \qquad \Pi_p = \langle \sigma_{xz} \rangle = -\Sigma \sin(Kz).$$

blue = Newtonian

red = polymers

green = Reynolds



Momentum budget:

$$\partial_z \Pi_r = \partial_z (\Pi_\nu + \Pi_p) + f_z,$$

$$SK + \nu U K^2 + \Sigma K = F.$$

$$\Pi_r \simeq 0$$

Low-Reynolds flow: kinetic energy budget

Energy input, viscous (Newtonian) and polymer energy dissipation:

$$\langle E \rangle = \frac{1}{2}\langle |\mathbf{u}|^2 \rangle, \qquad \epsilon_I = \langle f_i u_i \rangle, \qquad \epsilon_\nu = \nu \langle |\partial_j u_i|^2 \rangle, \qquad \epsilon_p = \langle \sigma_{ij} \partial_j u_i \rangle.$$

Kinetic energy budget:

$$\frac{d}{dt}\langle E \rangle = \varepsilon_I - \varepsilon_\nu - \varepsilon_p.$$

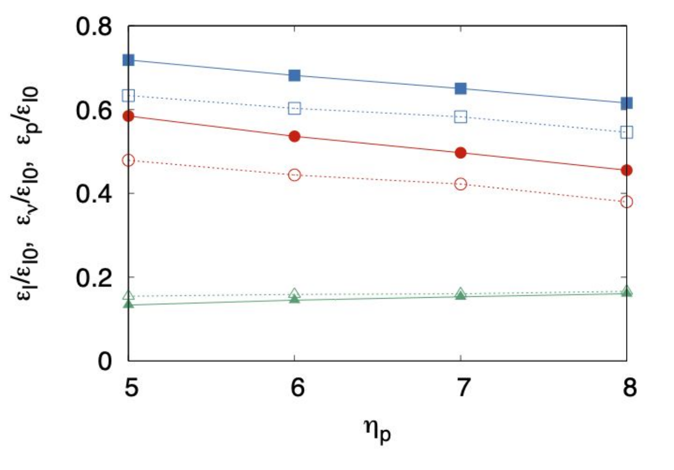
(statistical) Stationarity:

$$\frac{d}{dt}\langle E \rangle = 0 \quad \Rightarrow \quad \varepsilon_I = \varepsilon_\nu + \varepsilon_p.$$

blue = input

red = Newtonian

green = polymers

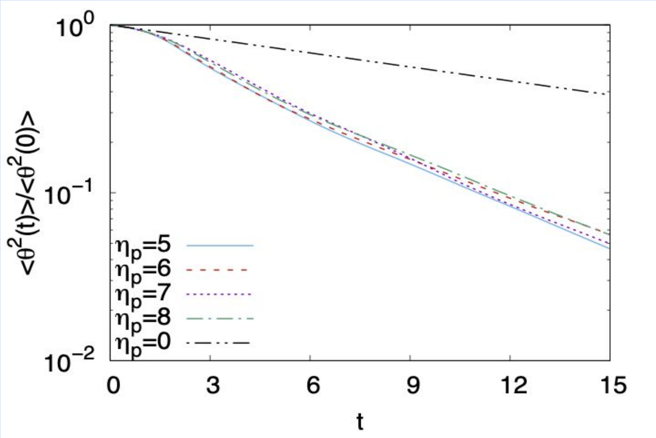


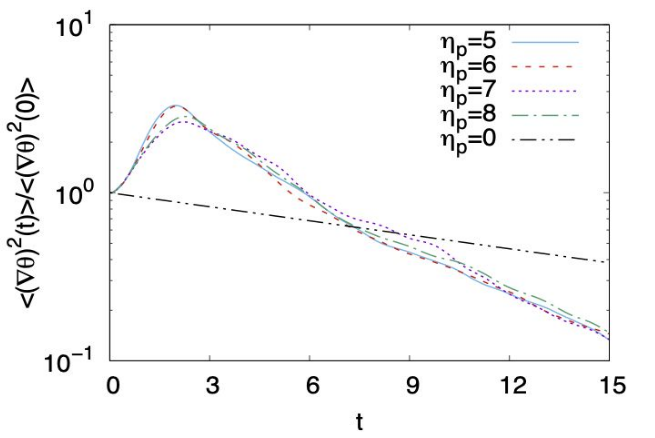
Low-Reynolds flow: mixing efficiency

Advection-diffusion equation for a passive scalar field:

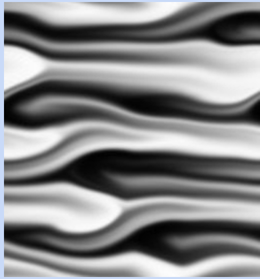
$$\partial_t \theta + u_k \partial_k \theta = D \nabla^2 \theta.$$

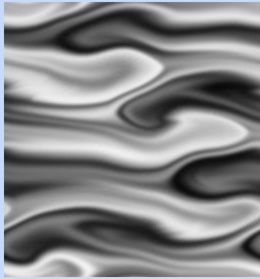
Mixing strongly enhanced compared to laminar flow:





Temporal decays of $\langle \theta^2 \rangle$ and $\langle |\nabla\theta|^2 \rangle$





Other activities

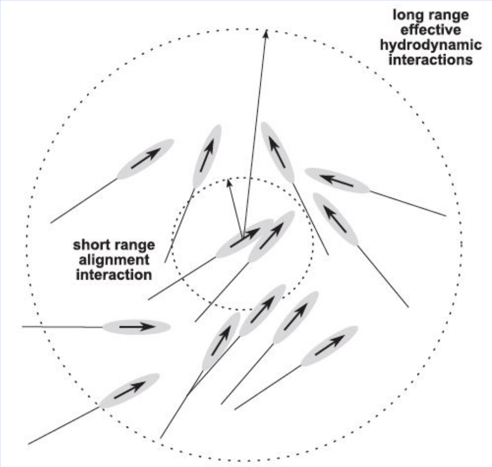
Writing of the paper regarding the master's thesis work:

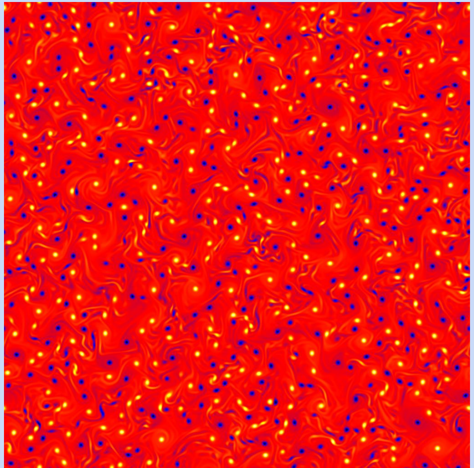
Puggioni, L., Kritsuk, A. G., Musacchio, S., & Boffetta, G. (2020). *Conformal invariance of weakly compressible two-dimensional turbulence.* Physical Review E, 102(2), 023107.

Future perspectives

- Writing of papers regarding rod-like polimers
- Towards the study of another kind of low-Reynolds chaotic flow: the “bacterial turbulence”:

$$(\partial_t + \lambda_0 \mathbf{P} \cdot \nabla)\mathbf{P} = -\nabla p - \alpha \mathbf{P} - \beta |\mathbf{P}|^2 \mathbf{P} + \Gamma_2 \nabla^2 \mathbf{P} + \Gamma_4 \nabla^4 \mathbf{P}.$$





Snapshot of 2D “vorticity” field